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FINAL REPORT
LONG-BOOM CONCEPTS

By John V. Coyner Jr., John M. Hedgepeth, and Harry D. Riead

Astro Research Corporation

1.0 SUMMARY

The objective of this study is to establish specifications and design criteria for large structures for future space missions, and to investigate design concepts for deployable mast structures with various guyline arrangements. As a baseline, a 100-meter-long structure has been selected for study which will be deployed from the STS Orbiter, capable of supporting a payload mass of 315 kg at the tip, and compatible with the STS Orbiter attitude-control system. This requirement is similar to the proposed Molecular Shield Vacuum Facility.

One of the primary concerns is the avoidance of deleterious interaction between the structure's flexibility and the attitude-control system. This study has produced new criteria for specifying the dynamic requirements for large structures. This criteria will furnish a powerful tool in assuring that conservative, but not over-conservative, dynamic designs are generated for a given set of requirements.

The results of the study indicate that guyline-stiffened structures provide a significantly higher stiffness-to-mass ratio than non-guyed structures. Guyline-stiffened structures also have the capability of positioning the payload in both rotation and translation directions.

2.0 INTRODUCTION

Advanced plans for space missions and payloads identify a variety of spacecraft as well as recurring requirements for certain ancillary equipment, notably deployable masts. For instance, plans through 1991 identify over 300 requirements for deployable masts. Plans for Shuttle payloads alone account for approximately 190 of those masts.

The mast requirements are diverse, and include masts for deploying:

1. Solar-cell arrays
2. Magnetometers
3. Antennas (tip-mounted antennas and the masts themselves)
4. Spectrometers for various gases, particles, and levels of radiation
5. Shadow shields
6. Energy collectors
7. Separable parts of the spacecraft
8. Gravity-gradient stabilization structures (tip-mounted masses and the masts themselves)
9. Remote manipulators
10. Molecular Shield Vacuum Facility (MSVF)

These diverse requirements will lead to a variety of mast designs, with some as long as 500 to 1000 meters and some as short as only a few meters. Technology presently exists for structures with lengths from 1 to 50 meters. Studies indicate that fabrication and/or assembly in space might be the best method for establishing structures with lengths greater than 150 meters. Therefore, this study will consider automatically deployable structures with lengths of 50 to 150 meters.

Proper structural design must begin with sound design requirements. For space operation, the most important of these deal with the dimensional stability and stiffness of the structure. There is, therefore, a need for rational criteria for determining the necessary stiffness.

The first part of this report presents a set of design criteria for the required stiffness for suspending masses from a spacecraft. Details of the derivation of these criteria are included in Appendix A. In addition to the dynamic requirements for the deployed structure, a preliminary set of requirements is generated which includes ground handling, launch environment, spacecraft accelerations, gravity-gradient loads, solar-pressure loads, precision, etc. (see Appendix B).

The second part of this report presents a description of three structural design concepts. These designs are compared relative to their ability to satisfy the dynamic requirements of a long structure attached to the STS Orbiter. Control requirements of the STS Orbiter are determined. The dynamic properties of the long structure are then specified from the criteria establishing the minimum safe stiffness. Designs for each concept are generated using the established criteria.

Recommendations are made regarding the technology which must be developed in order to realize these long structural systems and to explore their growth potentials.

3.0 REQUIREMENTS

The objective of this section is to generate specifications for large deployable structures. The specifications are derived from MSVF mission requirements, STS Orbiter launch-vehicle specifications, and ground handling and testing. These specifications provide a basis from which the designs for the candidate mast concepts are generated.

3.1 Dynamic Requirements

During this study, primary emphasis is placed on the dynamic characteristics of the mast and the payload system, and their relationship to the STS Orbiter attitude-control system dynamics. In other words, the development of criteria which ensure avoidance of deleterious interaction between the structural flexibility and the STS Orbiter attitude-control system. These criteria are applied as follows.

3.1.1 Derivation of Criteria

The idealized dynamic system is shown in Figure 1.

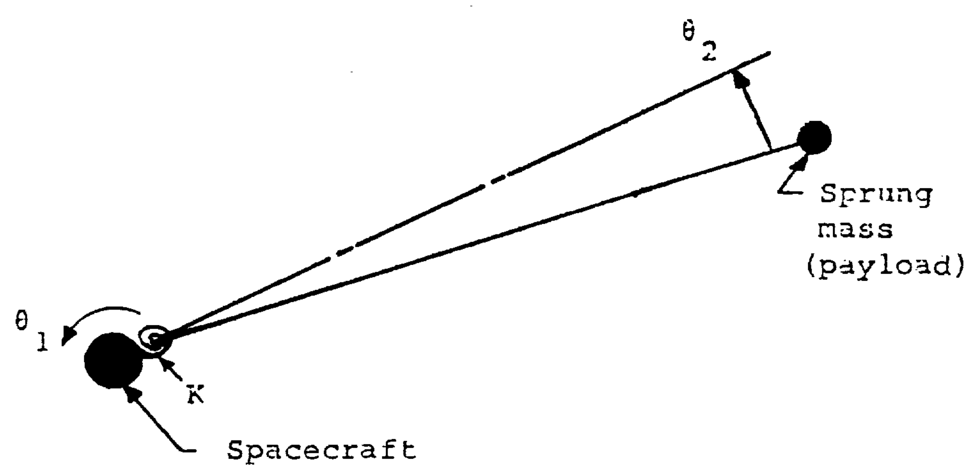


Figure 1.- Idealized system

SYMBOLS

I_1, I_2 = Moment of inertia of spacecraft and sprung mass, respectively.

θ_1, θ_2 = Angular displacements of spacecraft and sprung mass.

K = Spring stiffness

ω_n = Natural vibration frequency of sprung mass with spacecraft fixed ($\theta_1 = 0$).

ζ = Damping coefficient. Ratio of damping force to spring force for vibrations at frequency ω_n.

Let K_c be the control stiffness required to ensure sufficiently good pointing accuracy in the presence of static disturbances.

Define a control frequency ω_c as the frequency at which the spacecraft would oscillate with the control spring stiffness K_c and with the sprung mass rigidly attached. Thus

$$\omega_c = \sqrt{\frac{K_c}{I_1 + I_2}}$$

The ratio ω_n/ω_c is the basic design parameter. The results of the analysis in Appendix A define this ratio of ω_n/ω_c which will ensure avoidance of unstable interaction between structural flexibility and the attitude-control system.

The required values of ω_n/ω_c are shown in Figures 2a, 2b, and 2c as a function of inertia ratio for several representative values of ζ, ranging from 0.003 (representative of pure materials damping) to 0.1 (representative of "lossy" built-up structures). Results are presented for 0, 6, and 12 dB/octave roll-off.

Examination of the plots shows that the cantilever natural frequency required to avoid instability is not tremendously higher than the control frequency, even for very small damping. Note that the criterion is conservative and even smaller values of natural frequency may be adequate.

For large sprung masses, the required cantilever natural frequency can even be less than the control frequency. The resulting free-free natural frequency, however, is always higher than the control frequency.

For each value of damping coefficient ζ, there exists a value of inertia ratio for which the frequency ratio is a maximum. The maxima are plotted versus damping coefficient for the three roll-off cases in

Figure 3. This graph can be used for a very simple, albeit more conservative, criterion for preliminary design.

For a reasonable roll-off of 6 dB/octave and a damping coefficient of 0.01, a cantilever natural frequency of only 2.5 times the control frequency is conservatively adequate to preclude instability.

3.1.2 Application to MSVF Masts on STS Orbiter

To determine the mast properties, control-system frequencies have been obtained from an analysis of the dynamic behavior of a long boom mounted on the STS Oribter by Richard W. Faison of Langley Research Center. This analysis utilizes results from a simulation of the STS Orbiter attitude-control system and reveals that control frequencies in the range of 0.005 to 0.009 Hz can be expected when the vernier system is operated. A control frequency of 0.009 Hz is therefore selected for this study.

The following is a summary of the parameters used to determine ω_n and, thus, mast stiffness (EI).

ω_c	= 0.009 Hz
Roll-off	= 6 dB/octave
Tip mass	= 315 kg
Mast damping (ζ)	= 0.01*
I_2/I_1	= 0.5

Therefore, from Figure 2b,

$$\omega_n/\omega_c = 2.6$$

and

$$\omega_n = 0.024 \text{ Hz}$$

* Note:

The damping coefficient for Astromast is derived from test data of an 0.23-meter-diameter continuous-longeron Astromast. This mast was tested at Jet Propulsion Laboratory. It is assumed that the damping coefficient is not significantly affected by size of the mast. The damping coefficient of the column-supported-by-lattice-guy-lines mast is also assumed to be 0.01.

3.2 Complete Requirements

Many other pertinent requirements are considered during concept generation. For example, ground handling, launch environments, space environments, reliability, packaging envelopes, etc. A preliminary set of requirements has been generated and is presented in Appendix B.

4.0 STRUCTURAL CONCEPTS

Three different concepts of long mast structures are examined in this section. The mast is assumed to be 100 meters long with a payload tip mass of 315 kg. Primary emphasis is placed on satisfying the dynamic requirements which specify that the first-mode resonant frequency of the structural system is 0.024 Hz or greater. Launch loads and other requirements identified in Appendix B are not evaluated in this study.

By comparing the three designs against identical dynamic requirements, the stiffness-to-weight ratio and packaging envelope can be obtained for each design. The maximum critical bending-moment capability of each system will also be evaluated.

The first design evaluated is a non-guyed continuous-longeron Astromast (see Figure 4). The second system evaluated is a center column supported by two levels of guylines (see Figure 5). The third system evaluated is a center column supported by a lattice guyline system (see Figure 6).

4.1 Continuous-Longeron Astromast (non-guyed)

The Astromast is a linear lattice structure, or boom, which can be automatically deployed from, and retracted into, a compact stowage volume. There are two general types of Astromasts - continuous longeron and articulated longeron (see ref. 1). Only those with fiberglass continuous longerons are considered for this application. The manner in which this Astromast is deployed and retracted is illustrated in Figure 4. As shown, the lattice structure, composed of flexible fiberglass rods and shear-stiffened by diagonal cables, is retracted by forcibly twisting it about its axis. This twisting causes its battens (members perpendicular to boom axis) and longerons (members parallel to boom axis) to bend. This distortion of the boom and the mobility provided by the pivoted joints between the longerons and batten frames allows the boom to be retracted into the compact configuration as shown in Figure 4. The boom distortions are elastic in all stages so that deployment and retraction can be repeated.

This continuous-longeron type of Astromast can be designed to meet a wide variety of requirements, and it may or may not need a motor-driven canister for deployment and retraction, depending on the operational specifications of an application. The retracted length of this type of Astromast is typically one-fiftieth of its deployed length.

A canister deployed Astromast is selected for this application. This coilable Astromast is automatically deployed and stowed by a canister (see Figure 4). The upper portion of the canister is a rotatable, three-threaded nut. Lugs which protrude from the apices of each batten frame are engaged by the nut threads between stationary vertical rails. Thus engaged, the boom is deployed when the nut is rotated. Transition rails within the canister cause the mast to twist from its stowed, to its deployed configuration as the boom extends.

A canister for automatic deployment would be approximately two mast-diameters longer than the retracted mast length. When deployed from a canister, any deployed section of the mast has full strength. Therefore, the mast can be utilized structurally throughout deployment.

Table I summarizes the performance of the Astromast. The Astromast data presented in Table I are derived in Appendix C from equations presented in reference 1.

4.2 Center Column with Two-Level Guylines

This mast concept demonstrates the ability to produce a guyline-stiffened boom, while maintaining the simplicity of a single-point interface with the spacecraft or STS Orbiter, and providing a stowage envelope for the entire boom and guyline-support structure which is only slightly larger in diameter than the boom alone. This guyline-stiffening technique is also compatible with a variety of booms (e.g., Astromast, STEM, telescoping masts, etc.). Since the guylines are not required to interface with spacecraft hard-points, the angle and location of the guylines can be optimized for maximum overall boom strength and stiffness.

This concept consists of a central compression member which is stabilized by two sets of three guylines (see Figure 5). The central compression member (0.4-meter-diameter Astromast) deploys and pulls the upper and lower guyline sets from storage reels. This concept also has hinged guyline-support members which fold out during deployment and are supported by lower support cables. The guylines must maintain the required pre-tension during deployment in order for the mast to have the desired bending stiffness and strength throughout deployment.

The total length of the mast is 100 meters. The hinged guyline-support members intersect the mast 10 meters above the base. This fixes the package height at 10 meters and the length of the hinged guyline-support members at 20 meters (for a single hinge at the center of the member). The upper guyline set intersects the central column at the tip, and the lower guyline set intersects the central column 45 meters below the tip.

The upper and lower guylines are determined to be 0.64 cm by 0.013 cm steel tape. Typical upper guyline tensions are 50 N, while lower guyline tensions are 1.4 N. The size (cross-sectional area) of the guylines is a function of the required stiffness, while the guyline tension is determined from the lateral load capability of the mast.

The Astromast compression member is sized to carry the compression load of 151 N resulting from the guyline tension. An 0.4-meter-diameter Astromast is required.

The lower 10 meters of the central mast member is not supported by the guylines. This segment of the central member, therefore, must have adequate bending stiffness to ensure that the fundamental frequency of the structure will be greater than 0.0204 Hz. A 1.0-meter-diameter triangular cross-section truss constructed of graphite/epoxy tubular elements is determined to provide adequate bending stiffness to achieve the required mast stiffness. This lower bending section of the mast is the least efficient element for providing stiffness, since it is a bending member. Therefore, this section should be as short as possible.

Table I summarizes the performance of this concept. A detailed analysis is presented in Appendix D.

4.3 Column Supported by Lattice Guylines

This mast concept has a central compression member (BI-STEM) which is used to deploy and tension the members of a surrounding lattice structure (see Figure 6). The surrounding lattice structure has similar geometry to a continuous-longeron Astromast, except that the longerons are tension members (steel tapes) rather than the tension/compression members used in a continuous-longeron Astromast. Consequently, in order to develop bending strength, the longerons must be pre-tensioned by the central BI-STEM element. The design also has batten frames which are designed to provide lateral support for the BI-STEM as well as provide tension in the diagonals. Throughout deployment, the longitudinal guylines (longerons) must have the required pre-tension in order for the mast to have the desired bending strength and stiffness properties. Therefore, the BI-STEM must be deployed against an applied compressive load of 9 000 N. This load will necessitate a redesign of the BI-STEM deployment mechanism.

The cross-sectional diameter through the longerons is selected to be 1.12 meters, and the bay length (distance between batten frames) is determined to be 0.66 meter.

The cross-sectional area of the three longerons is determined from the requirement that the mast must have the same bending stiffness as a 1.12-meter continuous-longeron Astromast ($EI = 2.87 \times 10^6$ N-m^2). This requires a longeron cross-sectional area of 0.30 cm^2.

The tension in the longerons is determined from the requirement that the mast must have equal to, or greater, bending strength than the 1.12-meter continuous-longeron Astromast (1 500 N-m). The maximum bending strength of the mast is defined to be the bending moment at which an unloading longeron becomes slack or the stress in a loaded longeron exceeds 200 MPa. The required tension determined by this procedure is 3 000 N per longeron, which gives a critical bending moment of 2 600 N-m.

A stainless-steel BI-STEM is selected for the central element. The element is selected from the standard BI-STEM product line and the size is determined by the requirement that the element must support a compressive load of 9 000 N with a factor of safety of 1.5 on ultimate stress, crippling stress and Euler buckling over the 0.66-meter unsupported length between battens. This load is required to provide adequate tension in the longerons. The smallest standard BI-STEM which meets these requirements is a 3.4-cm-diameter BI-STEM.

The required cross-sectional area of the stainless-steel diagonals is determined from the requirement that the batten frames must supply adequate lateral support to the central BI-STEM element to ensure that the critical buckling mode will not involve lateral displacement of the batten frames. The diagonal cross-sectional area determined by this requirement is 0.054 cm^2.

The required tension in the diagonals is determined from the requirement that the structure must have adequate shear strength. Since tension in the diagonals is greatly affected by small errors in batten or diagonal length, excess diagonal tension must be provided to allow for manufacturing errors. The required diagonal tension is determined to be 30 N.

The diagonal tension of 30 N is produced by compression springs at the attachment points of the longeron and rigid-batten frames. The spring constants of these springs are set to provide adequate lateral stiffness for the central STEM.

The diagonal tension of 30 N results in a batten compression of 50 N. The batten members are graphite/epoxy tubes with d/t = 10 and are sized to have a factor of safety of 2 for Euler buckling under a compressive load of 50 N. The batten tube diameter is determined to be 1.4 cm. An alternative approach would be to use batten frames which are designed to buckle under a compressive load of 50 N. This approach, which is used in the continuous-longeron Astromast, would result in lighter batten frames.

The minimum package length is determined by the length of the stacked battens, plus 1.2 times the bay length for bay deployment, plus the length of the BI-STEM deployer. Since 152 battens are required (0.66-meter spacing) and the batten depth is 1.4 cm; the stack height is 2.2 meters. The length of the BI-STEM deployer is approximately 0.5 meters. The total package length is:

$$\text{Package length} = 2.2 + 0.5 + 1.2 \times 0.66$$
$$= 3.5 \text{ meters}$$

Table I summarizes the performance of this concept. A detailed analysis is presented in Appendix E.

4.4 Summary of Structural Concepts

All three concepts presented in this report satisfy the mission requirements of the Molecular Shield Vacuum Facility experiment. Table I summarizes the weights and stowage envelopes of each design. Through further design optimization both the weight and stowage envelope of each design can be improved; especially the center column with two-level guylines and the column supported by lattice guylines concepts.

The objective of the following sections is to discuss the performance, the inherent advantages and limitations, and the growth potential of each design.

MMARY

TWO-LEVEL GUYLINES	LATTICE GUYLINES
100 m	100 m
315 kg	315 kg
69 kg	207 kg
≥0.024	0.024 Hz
≥1 500 N-m	2 400 N-m
1.4 m diam x 10 m	1.2 m diam x 3.5 m
150 kg	150 kg

tain tension on the deploying longerons. Deployment requires linear motion supports to maintain tension on the longerons while the BI-STEM mechanism deploys the center column. Although synchronization is required, this deployment mechanism is considered to be of equivalent complexity and reliability as the continuous-longeron Astromast deployment mechanism.

Preliminary analysis indicates that this concept will have lower weight and higher strength than an Astromast with the same stiffness. Indications are that the growth potential for this design is comparable to the segmented-longeron Astromast.

Also, lateral stiffness is not provided by the bending stiffness of the mast, but by the axial stiffness of the guylines and bending stiffness of the short lower truss. Only a small percentage of the mast length (the lower non-deployable truss section) is loaded in bending. As shown in Table I, higher stiffness- and strength-to-weight ratios are achieved than with either of the other mast concepts. This design has excellent potential for satisfying future requirements for very long masts (greater than 150 meters) with low weight.

The primary disadvantage of this design is the long stowage length which is dictated by the length of the lower truss member. Design changes and optimization can reduce this length, however, preliminary estimates indicate that packaging lengths of 1.5 to 2 times the packaging length of the other two concepts can be expected.

The primary structural limitation of this design, and all other masts which have a single point attachment to the spacecraft, is associated with the flexibility of the attachment point to the spacecraft. The flexibility of the attachment point on the spacecraft can significantly degrade the effective stiffness of the deployed mast and therefore reduce the first-mode resonant frequency of the deployed system.

5.0 TECHNOLOGY DEVELOPMENT

The following areas of investigation must be pursued in order to achieve the goal of long, stiff, lightweight structures for space:

1. Improved definition of the dynamic properties of long structures, and the effects of hinges, joints, and materials on stiffness and damping.

2. Refinement of the criteria for avoidance of dynamic instability of the spacecraft attitude-control system for both linear and non-linear control systems.

3. Establishment of criteria for conservation of attitude-control system power usage on spacecraft with long flexible appendages.

4. Methods of acceptance and qualification testing and correlating analyses and ground testing with orbital performance.

5. Methods of packaging, guyline management, and deployment.

6. Determination of the effect of high-frequency broadband dynamic loads on long structures of low structural resonant frequencies.

7. Determination of the effect of thermal-induced perturbations on long structures.

8. Evaluation of effects of manufacturing tolerances, joint play, orbital thermal environment, and orbital loads on dimensional precision of long structures.

9. Determination of the limits on diameters and lengths to which automatically deployable masts can be extended and still remain compatible with launch-vehicle constraints and satisfy deployed orbital requirements.

The recommended program to accomplish the goal of a long, stiff, lightweight structure for space is to develop a meaningful space experiment utilizing the STS Orbiter as the launch vehicle and test facility. Since preliminary analyses show that structures greater than 100 meters long are feasible, the emphasis of the experiment would be verification of the dynamic analysis, verification of ground-testing techniques, determination of the sensitivity of the structure to various design parameters (joint play, guyline tension, active damping, payload positioning and orientation through the use of guylines, and thermal effects). Data derived from the experiment would be used to update analysis and testing techniques for predicting the performance of future long structures.

6.0 CONCLUDING REMARKS

1. The control-system stability criteria, to avoid instability, reveals that the required cantilever natural frequencies of long appendages on spacecraft are not tremendously higher than the control frequency, even for small damping. The criteria presented in this report are considered conservative, and even smaller values of structural natural frequencies may be adequate for stability.

2. Structural damping can significantly reduce the stiffness requirement and thus reduce the weight of long structures. Typically, a factor of 1.5 reduction in stiffness can be realized by increasing damping from 0.003 (representative of pure material) to 0.01 (representative of an Astromast), for a 6 dB/octave control frequency roll-off. However, a highly damped structure generally has large numbers of loose hinges and joints which are not compatible with precision structural requirements.

3. The stability criteria analysis presented herein should be refined and expanded to include determination of a criteria for conservation of attitude-control-system power and also to include non-linear control systems.

4. The most effective structure for providing bending stiffness is a structure which derives a major portion of its stiffness from tension and compression members and not bending members. Guylines provide this type of stiffness but have the inherent limitation of requiring multiple attachment points to the spacecraft. This significantly

complicates the structural interface and limits the variety of potential applications. A guyline-stiffened mast has been developed which maintains the versatility of the single-point attachment while providing a guyline-stiffened structure. This design has fold-out guyline-support arms to which the guylines are attached. The only section of the long mast that is loaded in bending is the lower 5 to 10 percent where the guyline supports are attached. Thus, a significantly more efficient stiffness-to-weight structure is achieved.

5. Preliminary results indicate that all three mast designs meet the stiffness requirements imposed by the STS Orbiter. Masts with lengths of 100 meters can be built to carry tip masses in the range of 300 to 500 kg. It is clear that this capability can be advanced to longer masts and heavier tip masses through further design optimization.

6. Because the guyline-stiffened mast, which has a single attachment point to the spacecraft, transmits all loads to the spacecraft through one point, the flexibility of that attachment point becomes critical. The effective stiffness of the mast can be significantly reduced if adequate stiffness is not provided by the spacecraft at that attachment point.

7. The design evaluation of the three mast designs has been limited. Further work is required to optimize these systems and to determine the limits to which these designs can be developed.

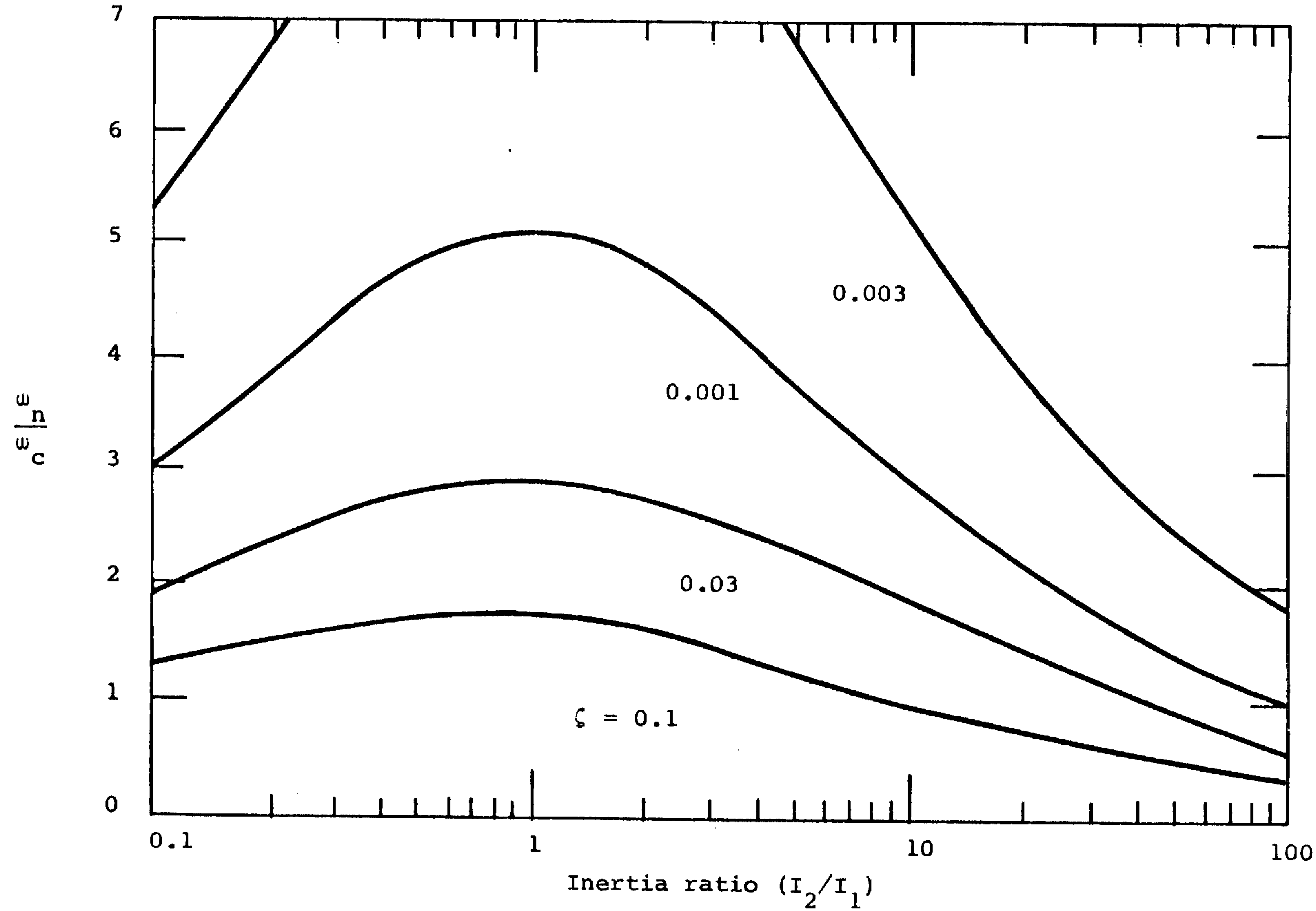


Figure 2a.- Required stiffness for stable control - no roll-off

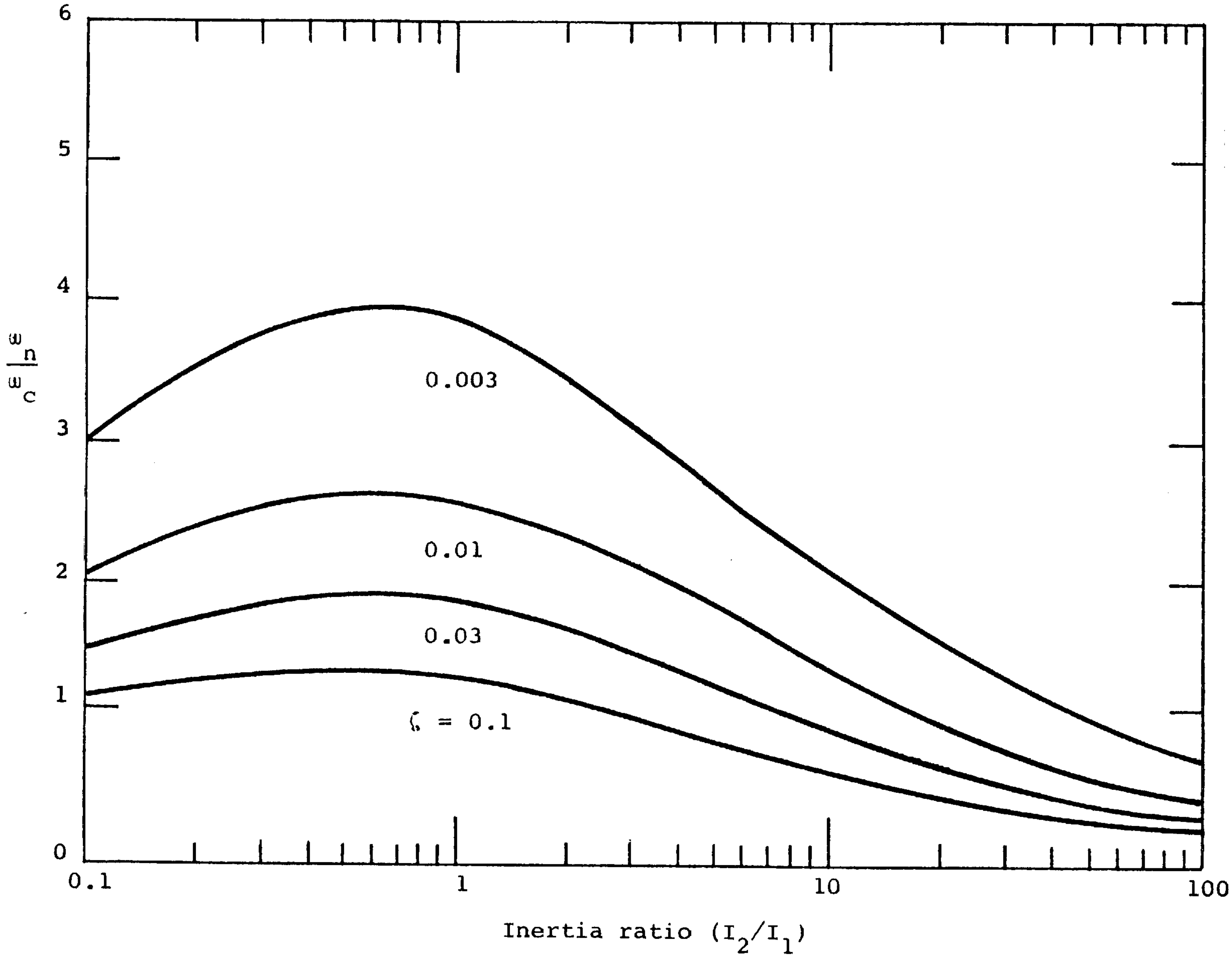


Figure 2b.- Required stiffness for stable control - 6 dB/octave roll-off

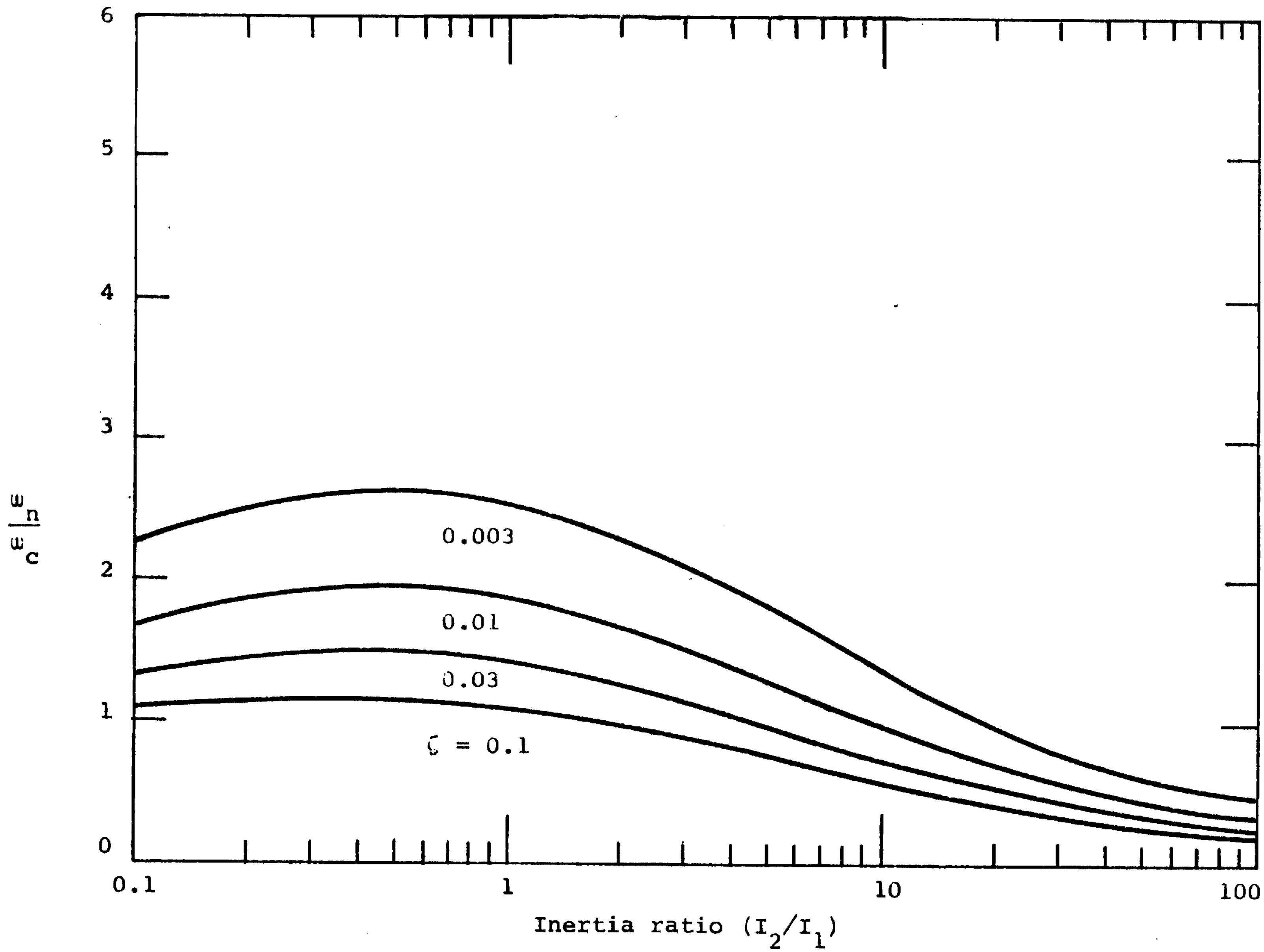


Figure 2c.- Required stiffness for stable control - 12 dB/octave roll-off

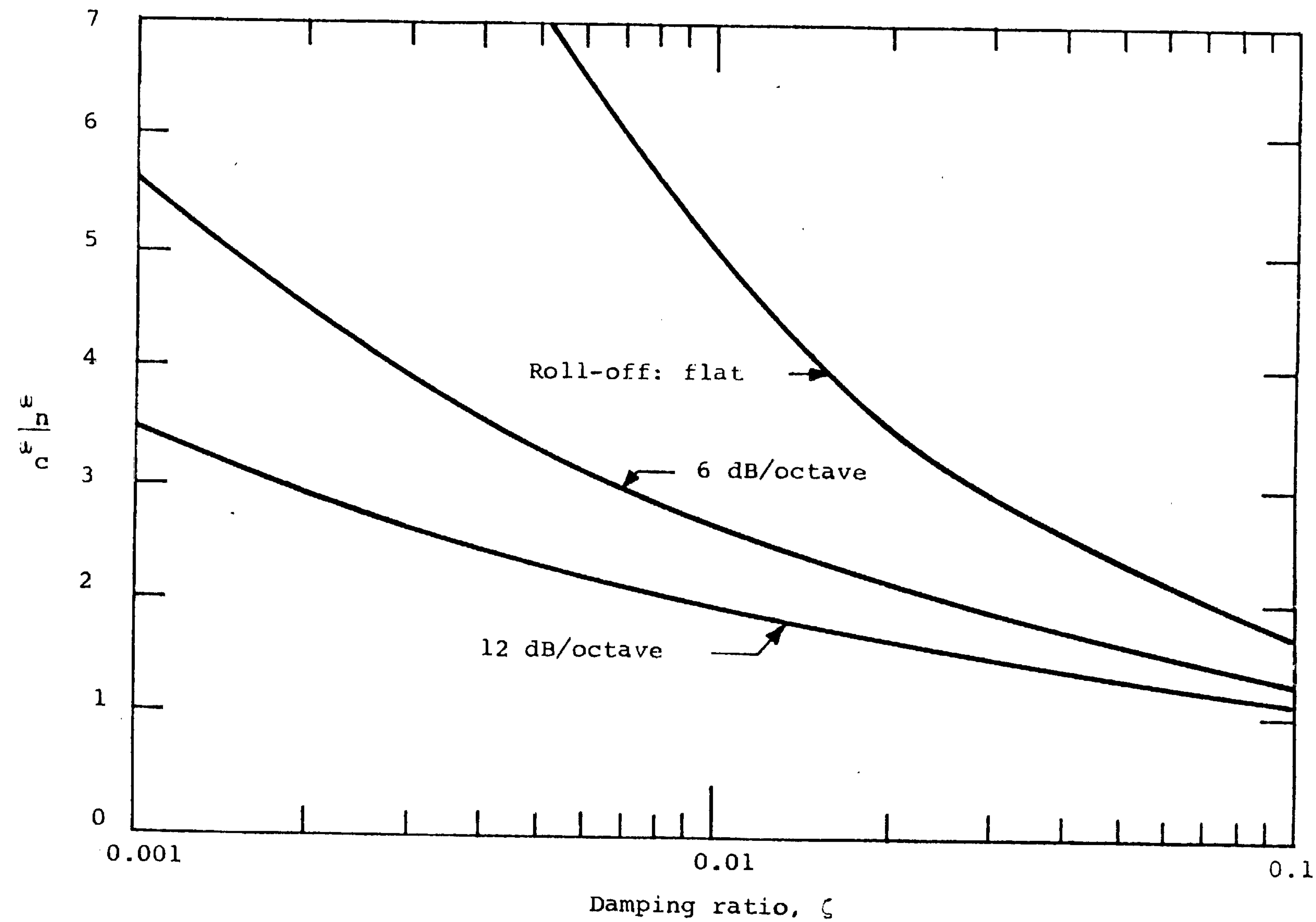


Figure 3.- Ratio of cantilever structural frequency to control frequency necessary to assure stability

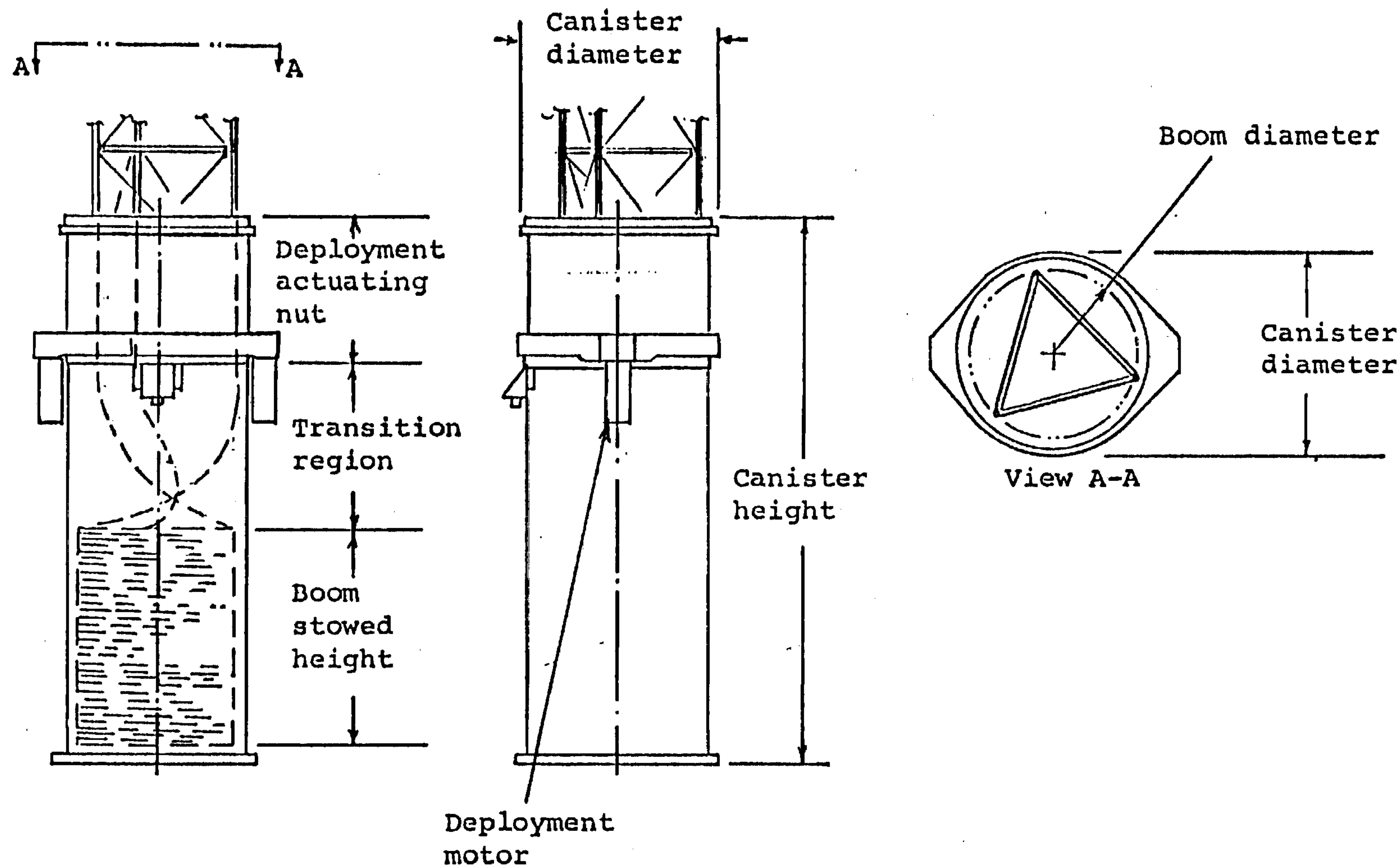


Figure 4.- Continuous-longeron Astromast (no guylines)

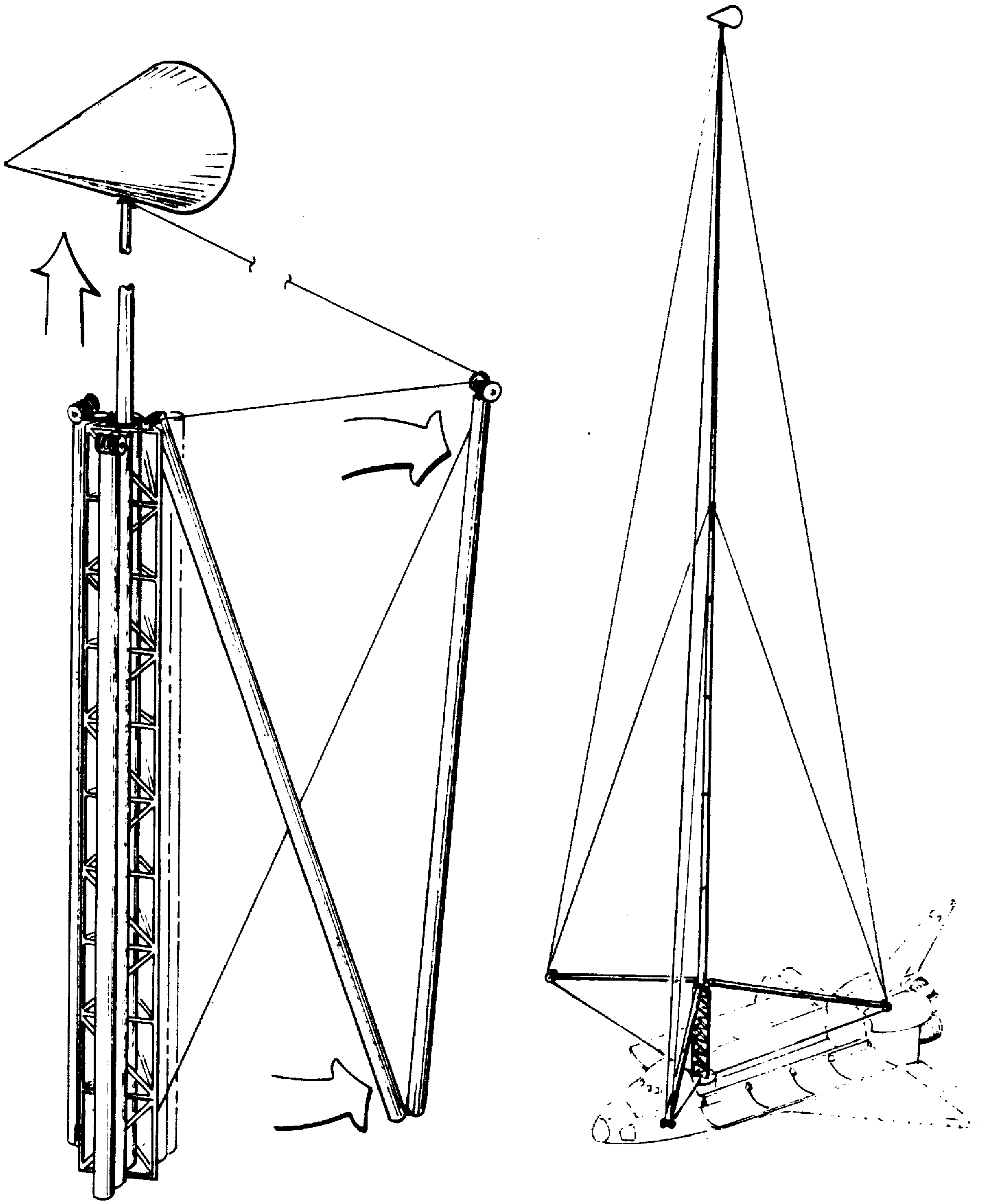


Figure 5. - Mast with central column and 2-level guylines

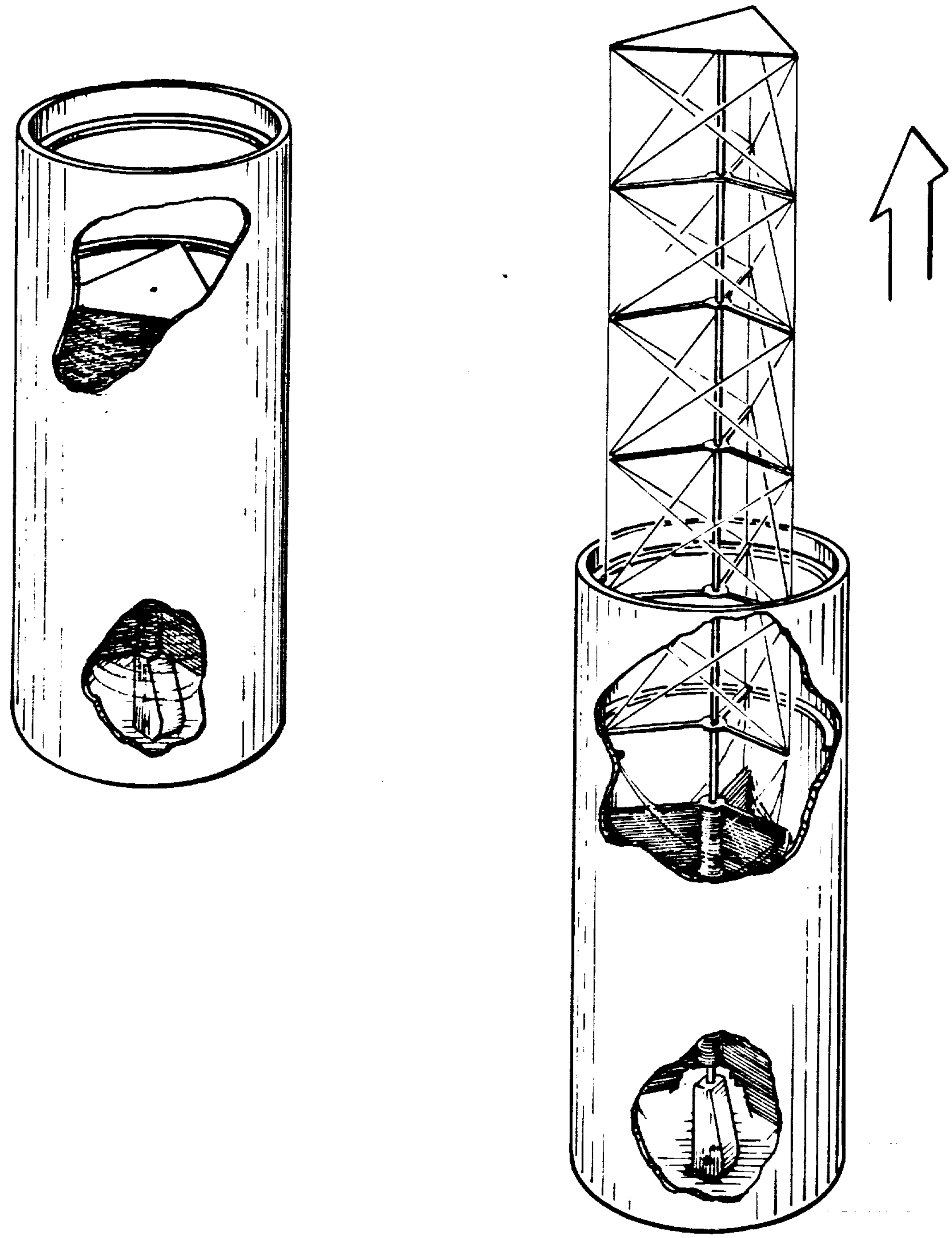


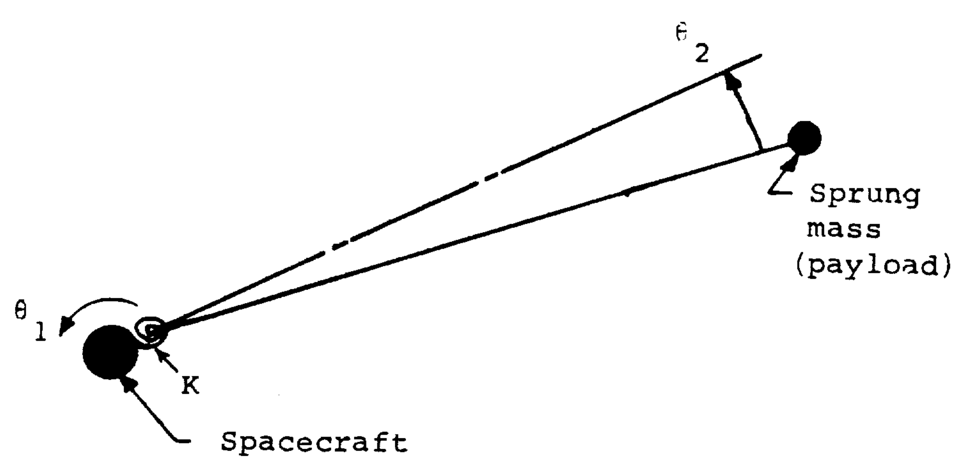
Figure 6. - Mast with center column and lattice guyline

APPENDIX A

DESIGN CRITERIA FOR FLEXIBLY SUSPENDED MASSES

The dynamic characteristics of long space structures are very important design considerations. Extendible spacecraft structures tend to be very flexible when deployed because of the low design loads and the desire for small weight and package volume. Numerous spacecraft have encountered difficulties ranging from excessive vibration, through control-system overloading, to outright catastrophic loss of attitude control because of unstable oscillation.

Experience has led to the ability to avoid these difficulties in new spacecraft by the application of criteria. These criteria are often not comprehensive, but function well enough for the current spacecraft with the probable cost of excessive weight.



I_1, I_2 = Moment of inertia of spacecraft and sprung mass, respectively.

θ_1, θ_2 = Angular displacements of spacecraft and sprung mass.

φ = Angular displacement of sprung mass with respect to spacecraft.

Q_1, Q_2 = Torques on spacecraft and sprung mass.

K = Spring stiffness.

K_c = Control stiffness.

ω_n = Natural vibration frequency of sprung mass with spacecraft fixed ($\theta_1 = 0$).

ω_c = Control frequency.

ζ = Damping coefficient. Ratio of damping force to spring force for vibrations at frequency ω_n.

Q_c = Control torque applied to spacecraft.

Q_{1e}, Q_{2e} = External torques

The equilibrium equations of motion are (see sketch above)

$$I_1 \ddot{\theta}_1 + K(\theta_1 - \theta_2) + \frac{\zeta K}{\omega_n}(\dot{\theta}_1 - \dot{\theta}_2) = Q_1 \tag{1}$$

$$I_2 \ddot{\theta}_2 + K(\theta_2 - \theta_1) + \frac{\zeta K}{\omega_n}(\dot{\theta}_2 - \dot{\theta}_1) = Q_2 \tag{2}$$

where the torques are

$$Q_1 = Q_{1e} + Q_c$$

$$Q_2 = Q_{2e}$$

The control torque Q_c is taken to be a function of θ_1 only.

Rewriting the equations in terms of the relative displacement $\varphi = \theta_2 - \theta_1$ and rearranging yields

$$(I_1 + I_2)\ddot{\theta}_1 = -I_2\ddot{\varphi} + Q_c(\theta_1) + Q_{1e} + Q_{2e} \tag{3}$$

$$I_2\ddot{\varphi} + \frac{\zeta K}{\omega_n}\dot{\varphi} + K\varphi = -I_2\ddot{\theta}_1 + Q_{2e} \tag{4}$$

Take Laplace transforms, designate the transform variable as p and the transformed quantities with bars, and set

$$\overline{Q}_c = -K_c R(p)\,\overline{\theta}_1 \tag{5}$$

where K_c is the control stiffness required to ensure sufficiently good pointing accuracy in the presence of static disturbances.

The function R(p) represents the dynamic characteristics of the control system. It is equal to unity for small p.

Define a control frequency ω_c as the frequency at which the spacecraft would oscillate with the control spring stiffness K_c and with the sprung mass rigidly attached. Thus

$$\omega_c = \sqrt{\frac{K_c}{I_1 + I_2}} \tag{6}$$

Equations (3) and (4) are then written

$$\bar{\theta}_1 = G_R(p)\bar{\theta}_1 - \frac{I_2}{I_1 + I_2}\bar{\varphi} + \frac{\bar{Q}_{1e} + \bar{Q}_{2e}}{(I_1 + I_2)p^2} \tag{7}$$

$$\bar{\varphi} = G_S(p)\left(\frac{-p^2\bar{\theta}_1}{\omega_n^2} + \frac{\bar{Q}_{2e}}{\omega_n^2 I_2}\right) \tag{8}$$

where

$$G_R = -\frac{\omega_c^2 R(p)}{p^2} \tag{9}$$

$$G_S = \frac{\omega_n^2}{p^2 + \zeta\omega_n p + \omega_n^2} \tag{10}$$

in which

$$\omega_n = \sqrt{\frac{K}{I_2}} \tag{11}$$

Combining gives

$$\bar{\theta}_1 = \left[G_R(p) + \frac{I_2 p^2}{(I_1 + I_2)\omega_n^2} G_S(p)\right] \bar{\theta}_1 + \frac{\bar{Q}_{1e} + \bar{Q}_{2e}}{(I_1 + I_2)p^2} - \frac{\bar{Q}_{2e}}{(I_1 + I_2)\omega_n^2} G_S(p) \qquad (12)$$

Note that G_S is the dynamic response function of a damped oscillator with frequency ω_n. In actuality the spacecraft sprung-mass combination has a natural frequency of

$$\omega_f = \sqrt{1 + \frac{I_2}{I_1}}\, \omega_n \qquad (13)$$

The mode of vibration associated with this frequency is such that $I_1\theta_1 + I_2\theta_2$ is zero. It is useful to re-cast the equations so that this free-free frequency is involved.

Let θ be the average motion. Thus

$$\theta = \frac{I_1\theta_1 + I_2\theta_2}{I_1 + I_2} \qquad (14)$$

Then the equations in transform space can be rewritten as

$$\bar{\theta} = G_R(p)\left(\bar{\theta} - \frac{I_2}{I_1 + I_2}\varphi\right) + \frac{\bar{Q}_{1e} + \bar{Q}_{2e}}{(I_1 + I_2)p^2} \qquad (15)$$

$$\bar{\varphi} = G_f(p)\left(-\frac{p^2\bar{\theta}}{\omega_n^{\,2}} + \frac{\bar{Q}_{2e}}{\omega_n^{\,2} I_2}\right) \tag{16}$$

where G_R is as before and

$$G_f = \frac{\omega_n^{\,2}}{\frac{1}{1+\frac{I_2}{I_1}}\,p^2 + \zeta\,\omega_n\,p + \omega_n^{\,2}} \tag{17}$$

Combining gives

$$\bar{\theta} = G_R(p)\left[1 + \frac{I_2 p^2}{(I_1 + I_2)\omega_n^{\,2}}\,G_f(p)\right] + \frac{\bar{Q}_{1e} + \bar{Q}_{2e}}{(I_1 + I_2)p^2} - \frac{\bar{Q}_{2e}}{(I_1 + I_2)\omega_n^{\,2}}\,G_R(p)G_f(p) \tag{18}$$

Note that the dynamic response of the free-free flexible mode is exhibited in G_f. The format of Equations (12) and (18) will be useful in estimating the criteria for avoiding instability.

The response of the system to external excitation is

$$\bar{\theta}_1 = \frac{\bar{Q}_{1e} + \bar{Q}_{2e}}{p^2(I_1 + I_2)}\;\frac{\frac{1}{G_S G_R}}{\frac{1}{G_R G_f} - \frac{1}{G_S}} - \frac{Q_{2e}}{\omega_n^{\,2}(I_1 + I_2)}\;\frac{\frac{1}{G_R}}{\frac{1}{G_R G_f} - \frac{1}{G_S}} \tag{19}$$

$$\bar{\theta} = \frac{\bar{Q}_{1e} + \bar{Q}_{2e}}{p^2(I_1 + I_2)} \frac{\frac{1}{G_f G_R}}{\frac{1}{G_R G_f} - \frac{1}{G_S}} - \frac{Q_{2e}}{\omega_n^{\ 2}(I_1 + I_2)} \frac{1}{\frac{1}{G_R G_f} - \frac{1}{G_S}} \tag{20}$$

$$\bar{\varphi} = -\frac{\bar{Q}_{1e} + \bar{Q}_{2e}}{\omega_n^{\ 2}(I_1 + I_2)} \frac{\frac{1}{G_R}}{\frac{1}{G_R G_f} - \frac{1}{G_S}} + \frac{Q_{2e}}{I_2 \omega_n^{\ 2}} \frac{\left(\frac{1}{G_R} - 1\right)}{\frac{1}{G_f G_R} - \frac{1}{G_S}} \tag{21}$$

Equation (19) will be useful in determining the effect of flexibility on control-power usage.

Dynamic Stability

Classical theory of the dynamic stability of linear systems shows that a system is stable if, and only if, there exist no roots with the positive real parts of the characteristic equation of the homogeneous system. In the present instance, the characteristic equation can be expressed as

$$1 = G_R(p) + \frac{I_2 p^2}{(I_1 + I_2)\omega_n^{\ 2}} G_S(p) \tag{22}$$

or, alternatively as

$$1 = G_R(p) \left[1 + \frac{I_2 p^2}{(I_1 + I_2)\omega_n^{\ 2}} G_f(p) \right] \tag{23}$$

The Nyquist criterion is a convenient method of determining the number of roots lying in the right-hand half plane. Thus, if the pure imaginary axis is traversed in the p-plane from $-i\infty$ to $+i\infty$, then the number of roots of the equation

$$1 = Z(p)$$

lying in the right-hand half plane is equal to the number of clockwise encirclements of the point $Z = 1$ in the Z-plane.

In the present case, there are presumed to be no unstable roots of the equation

$$1 = G_R(p)$$

otherwise the spacecraft would be unstable even if the sprung mass were rigid. The Nyquist mapping of the function $G_R(p)$ therefore must not produce any clockwise encirclements of the point 1. The question is: Does the flexibility-caused modification to the right-hand side of Equations (22) or (23) shift the mapped contour so that clockwise encirclements are introduced? This is the question to be addressed.

First, note that in Equation (22), for $\rho = i\omega$, the expression

$$\frac{I_2 p^2}{(I_1 + I_2)\omega_n^2} G_S(p) = \frac{I_2}{I_1 + I_2} \frac{p^2}{p^2 + \zeta \omega_n p + \omega_n^2}$$

is small for ω small and approaches

$$\frac{I_2}{I_1 + I_2} < 1$$

for ω large. Since the shift is small for small ω and since $|G_R(p)| \ll 1$ for large ω, any change in encirclements cannot result in these ranges. A similar conclusion can also be made for the alternative formulation, Equation (23).

Therefore, only two possibilities exist in which there is danger of producing a shift sufficient to cause an encirclement:

a) For $\omega \cong \omega_c$, because $G_R(p) \cong 1$ there.

b) For $\omega \cong \omega_n$ (Equation (22)) or ω_f (Equation (23)), because the modification is so large at the resonant condition.

Now, in the neighborhood of $p = i\omega_c$, the function G_R must have a positive imaginary part. Otherwise, there would be a clockwise encirclement produced. The added function in Equation (22) is

$$-\frac{I_2\omega_c^{\,2}}{(I_1 + I_2)\omega_n^{\,2}\left[\omega_n^{\,2} - \omega_c^{\,2} + i\zeta\,\omega_n\omega_c\right]}$$

or, for Equation (23)

$$\frac{I_2}{I_1 + I_2}\,\frac{\omega_c^{\,2}}{\omega_n^{\,2}}\,\frac{\omega_c^{\,2} - \omega_n^{\,2} + i\,\zeta\,\omega_n\omega_c}{\left(\omega_c^{\,2} - \omega_n^{\,2}\right)^2 + \zeta^2\omega_n^{\,2}\omega_c^{\,2}}$$

which also has a positive imaginary part. Therefore, the influence of flexibility is to move the contour away from the point 1. Hence, possibility (a) does not cause any difficulty. The same conclusion is reached for the alternative Equation (23).

Now, to examine possibility (b), consider that the flexible portion is lowly damped. A <u>sufficient</u> condition for there to be no encirclements produced near the resonant frequency is that the absolute value at resonance of the function on the right-hand side is less than unity. Therefore, a conservative criterion can be established by setting

$$1 > \left|\left(\frac{\omega_c}{\omega_n}\right)^2 \quad R(i\omega_n) - \frac{I_2}{I_1 + I_2}\,\frac{1}{i\zeta}\right| \tag{24}$$

for Equation (22) or, alternatively

$$1 > \left(\frac{\omega_c}{\omega_n}\right)^2 \frac{\left|R(i\omega_f)\right|}{1 + \frac{I_2}{I_1}}\left|1 - \frac{I_2}{I_1}\,\frac{1}{i\,\zeta\sqrt{1 + \frac{I_2}{I_1}}}\right| \tag{25}$$

for Equation (23)

For $R(i\omega_n) = R(i\omega_f) = 1$, the two alternative criteria become, respectively

$$\left(\frac{\omega_n}{\omega_c}\right)^4 > \frac{\zeta^2}{\zeta^2 - \left(\frac{I_2}{I_1 + I_2}\right)^2} \tag{26}$$

$$\left(\frac{\omega_n}{\omega_c}\right)^4 > \frac{1}{\left(1 + \frac{I_2}{I_1}\right)^2} \left[1 + \left(\frac{I_2}{I_1\zeta}\right)^2 \frac{1}{1 + \frac{I_2}{I_1}}\right] \tag{27}$$

Since these arise from sufficient conditions, either criterion is adequate. Clearly Equation (26) is more stringent than Equation (27). It therefore will be discarded. There are surely more relaxed, but still adequate, criteria than that of Equation (27); however, the search for these is left to the future.

Equation (25) is therefore adopted for the present criterion. Written for more general function R(p) it becomes

$$\left(\frac{\omega_n}{\omega_c}\right)^2 > \frac{\left|R(i\omega_f)\right|}{\left(1 + \frac{I_2}{I_1}\right)^{3/2}} \frac{I_2}{I_1\zeta} \sqrt{1 + \left(\frac{I_1}{I_2} + \frac{I_2}{I_1}\right)\zeta^2} \tag{28}$$

Control systems normally employ a roll-off of the gain for frequencies greater than the control frequency. This roll-off can be represented by appropriate choice of the function R.

Assume that

$$\begin{aligned} \left|R(i\omega)\right| &= 1 \qquad , \ \omega < \omega_c \\ &= \left(\frac{\omega_c}{\omega}\right)^r \quad , \ \omega > \omega_c \end{aligned}$$

where:

r = 0 for a flat gain

= 1 for 6 dB/octave roll-off

= 2 for 12 dB/octave roll-off

The resulting criterion is,

$$\left(\frac{\omega_n}{\omega_c}\right)^{2+r} = \frac{1}{\left(1 + \frac{I_2}{I_1}\right)^{\frac{3+r}{2}}} \frac{I_2}{I_1 \zeta} \sqrt{1 + \left(\frac{I_1}{I_2} + \frac{I_2}{I_1}\right)\zeta^2} \tag{30}$$

APPENDIX B

PRELIMINARY SPECIFICATION

1.0 SPECIFICATION OUTLINE AND DESCRIPTION

The following is a preliminary specification for a long mast to be deployed in space.

1.1 Mission Requirements

This section defines the launch vehicle, orbit, mission lifetime, and mission activities that take place during the orbital lifetime. The selected mission for this study of mast concepts is the Molecular Shield Vacuum Facility (MSVF). The launch vehicle is the STS Orbiter with a 100-nautical-mile orbit.

1.2 Functional Description

This section defines the orbital configuration of the vehicle. The selected mast configuration is a 100-meters in length with a payload at the tip, which will deploy from the STS Orbiter. Retraction capability of the mast is required.

1.3 Mechanical Performance

1.3.1 Payload System (s) Support

This section defines the payload for which the deployable structure provides positioning and structural support. Included is a description of the stowed and deployed geometry, payload mass, and location of deployed payload. For this study, a 315 kg payload will be deployed 100 meters from the STS Orbiter. The stowed envelope of the mast will be minimized and must be compatible with the STS Orbiter envelope. The mast structure will be designed to meet all requirements with minimum weight. No weight budget is identified at this time.

1.3.2 Precision

The mast will provide payload mounting points that are accurate and stable throughout the mission life and identified environments. Lateral position, axial position, and slope will be defined. All potential errors will be considered, including fabrication errors, repeatability errors, thermal distortion, material degradation, wear, and errors produced from manufacturing and testing in 1 g and operation in 0 g environment.

1.3.3 Structural Requirements

This section defines the structural requirements for ground-handling loads, launch loads, deployment loads, orbital loads, stowed natural frequency and deployed dynamics. The following paragraphs discuss the various requirements.

1.3.3.1 Design Factors of Safety

Standard factors of safety for spacecraft systems will be used:

F. S. = 1.25 on yield stress

F. S. = 1.50 on ultimate stress

1.3.3.2 Structural Requirements (Stowed Configuration)

These requirements are derived from standard ground-handling load requirements and launch-vehicle environments. The mast will be capable of surviving these environments without structural degradation or permanent deformation.

1.3.3.2.1 Minimum Vibration Frequency

To minimize launch dynamic loads, the mast structure and payload will have lateral and longitudinal frequencies above 10 Hz.

1.3.3.2.2 Environmental Limit Loads

1.3.3.2.2.1 Ground-Handling Loads

The mast in the stowed configuration will be capable of surviving the following ground-handling loads, (reference 2).

A. Shock

1. The shock environments experienced during handling are 20 g terminal-sawtooth shock pulse of an 11 millisecond duration in each of 6 axes.

B. Acceleration

(Hoisting loads) 2 g vertical within a cone angle of ± 20 degrees.

C. Vibration

The vibration spectrum is a minimum of four (4) sweeps at 1/2 octave per minute at the following levels (sinusoidal motion).

2 - 5 Hz	@	25.4 mm double-amplitude
5 - 26 Hz	@	1.3 g peak
26 - 50 Hz	@	0.91 mm double-amplitude
50 - 100 Hz	@	5 g peak

1.3.3.2.2.2 Launch Environment

Maximum vibrational loads occur during the launch and transonic periods of flight. The input to the mast is divided into two discrete regimes: low-frequency sinusoidal-vibration excitation, which tends to control the design of the major portions of the structure; and high-frequency broad-band random-vibration excitation, which tends to control the design of components. Rigorous definition of accelerations at the interfaces of subsystems will be difficult to determine during this study because payload design accelerations at their attachment points will vary with payload mass, and because subsystem design accelerations depend on the spacecraft design and the attachment method between spacecraft and subsystem. Therefore, conservative design limit load factors are generated that are representative of existing flight-program launch loads for subsystems.

1.3.3.2.2.2.1 Launch Acceleration

The following conservative subsystem limit load factors are used:

Direction	Acceleration
X	± 4 g
Y	± 4 g
Z (thrust axis)	± 10 g

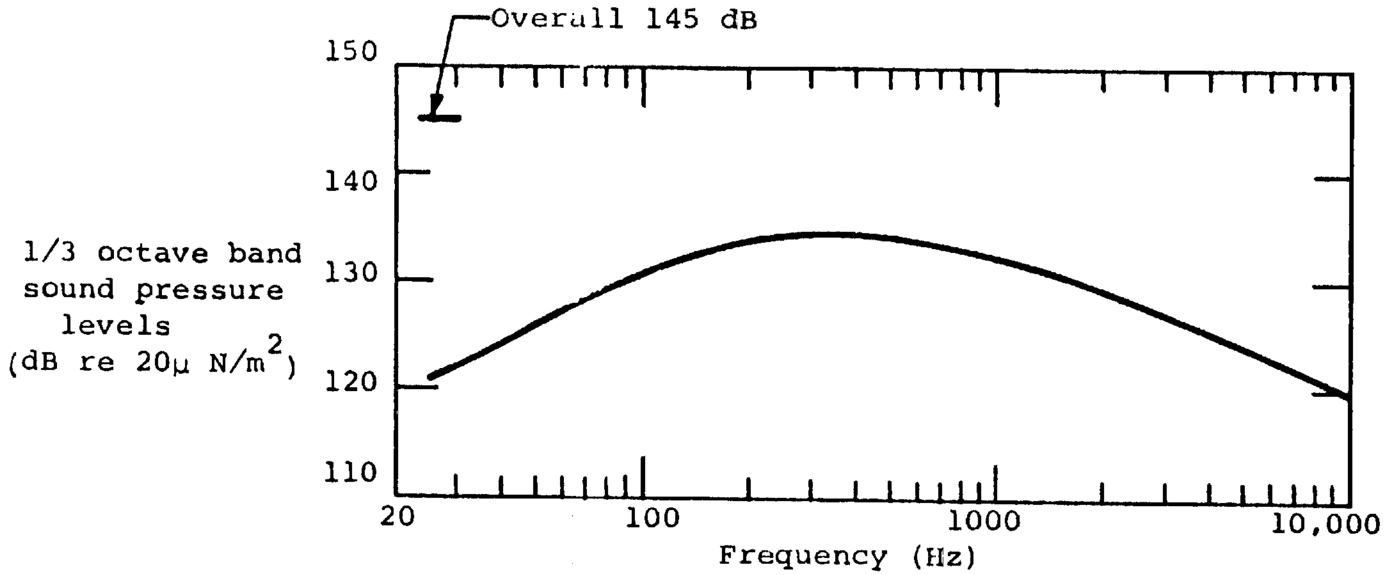
These loads are to be applied simultaneously.

1.3.3.2.2.2.2 Acoustics

Maximum acoustic noise levels occur at launch (due to engine noise) and in the transonic region (due to aerodynamic noise created by boundary-layer fluctuation). Typical durations of exposure are one minute. Overall sound pressure environment for the STS Orbiter is 145 dB. Figure B-1 defines the acoustic environments for the STS Orbiter, (reference 2).

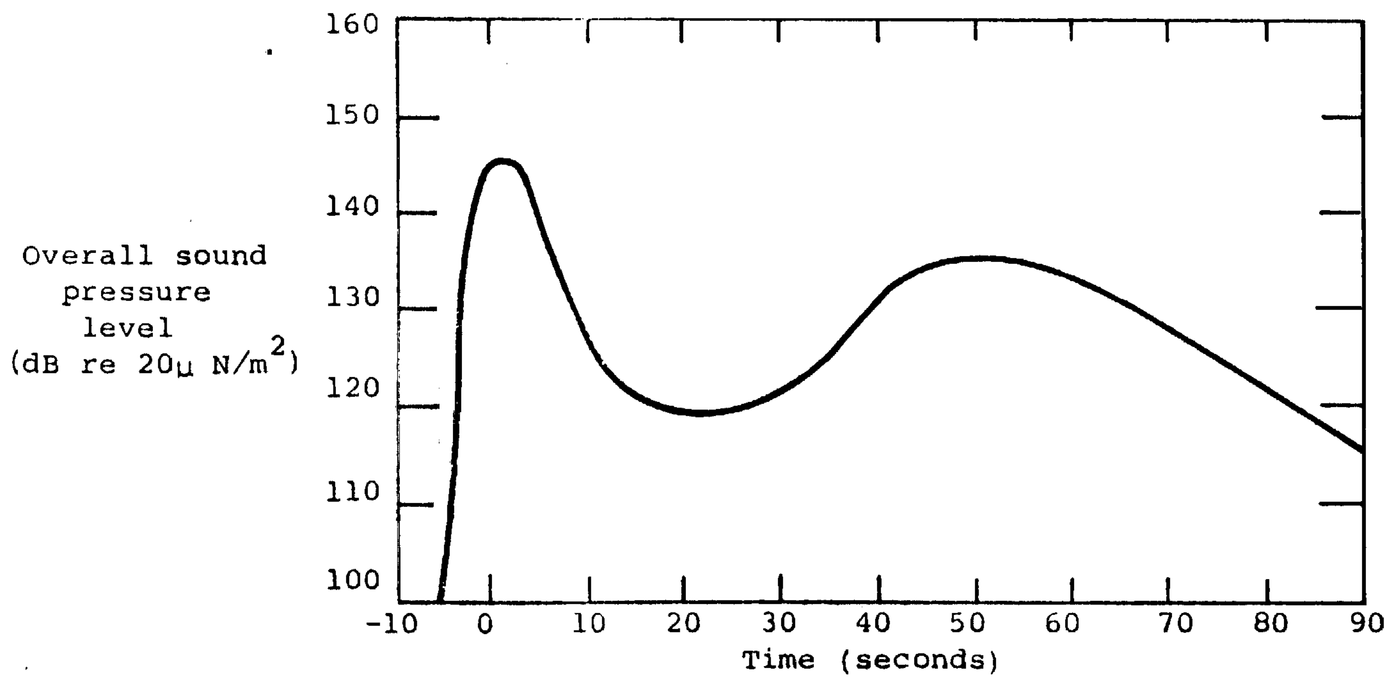
1.3.3.2.2.2.3 Random Vibration

It is assumed for preliminary design analysis that the loads produced by the random-vibration environment are accounted for in the specified design limit load factors. The random vibration environment levels for the STS Orbiter are presented in Figure B-2. The duration of these loads is approximately 30 seconds, (reference 2).



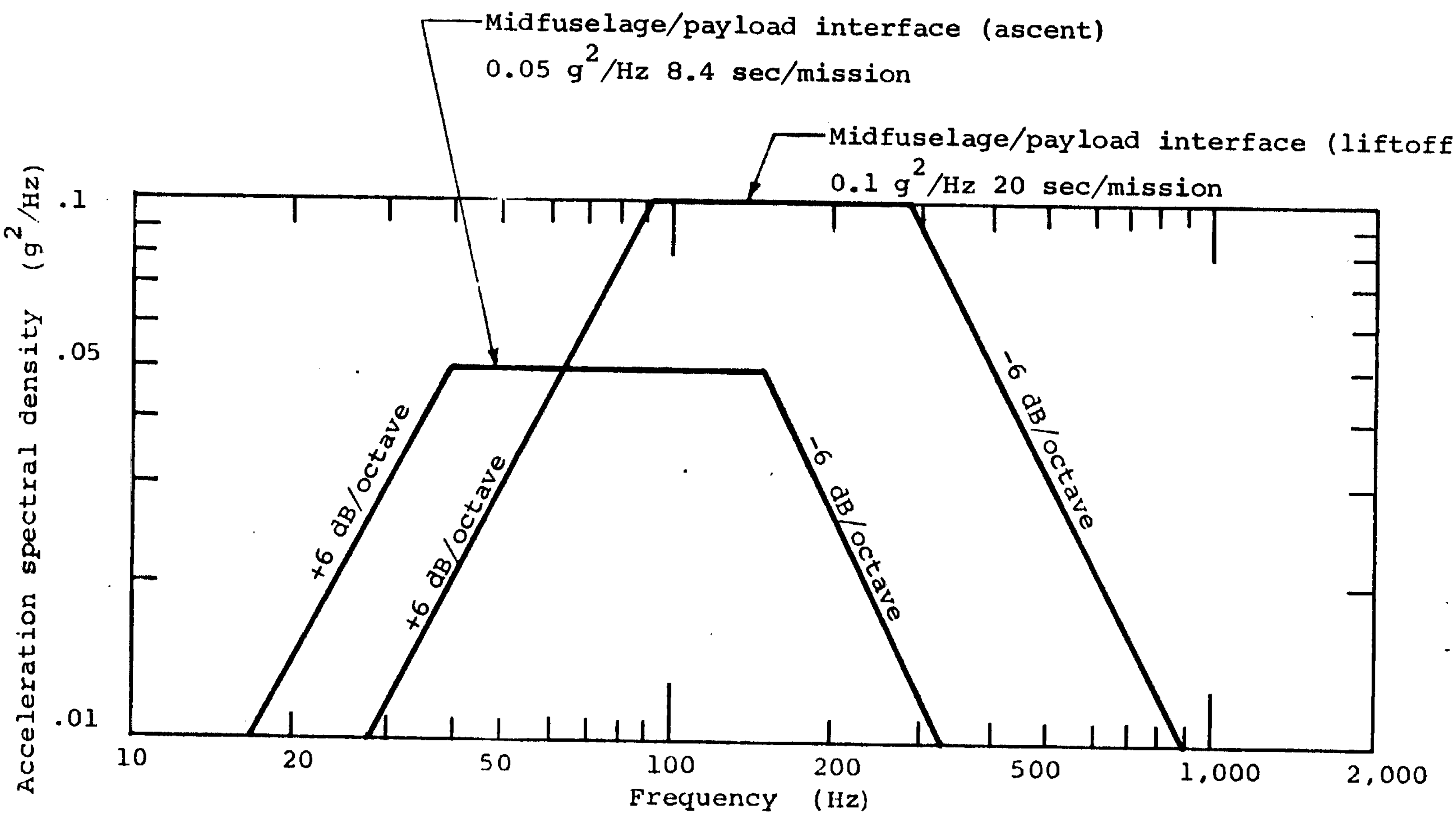
a.- Analytical predictions maximum Orbiter payload bay internal acoustic spectra.

Figure B-1.- Acoustic environment for the STS Orbiter



b.- Orbiter payload bay internal acoustic time history

Figure B-1.- Concluded



Note: Actual vibration input to payloads will depend on transmission characteristics of midfuselage-payload support structure and interactions with each payload's weight, stiffness, and c.g.

Figure B-2. - Random vibration at payload midfuselage interface for STS Orbiter.

1.3.3.2.2.2.4 Shock

Shock (transient vibration) is produced by two sources, the STS Orbiter and pyrotechnic devices. The STS Orbiter shock loads are accounted for in the swept sinusoidal-vibration environment.

Shock levels resulting from pyrotechnic devices are a function of their distance from the subsystem being considered. Figure B-3 defines the shock sprectrum of typical pyrotechnic devices and Figure B-4 defines attenuation versus distance from the pyrotechnic device.

1.3.3.2.2.2.5 Pressure

The STS Orbiter is vented during launch. The pressure inside the launch vehicle closely follows the flight atmospheric pressures. The payload-pressure time history for the vehicle is shown in Figure B-5, (Reference 2). The worst-case is approximately 1.73 KPa/sec. A conservative 21 KPa/sec is assumed.

1.3.3.3 Structural Requirements (Orbital Configuration)

Three orbital conditions are considered:

1. Stowed Configuration - Launch restraint released
2. Deployment - During extension and retraction
3. Orbital Configuration - Fully deployed and locked

1.3.3.3.1 Stowed

The accelerations prior to deployment are assumed to be identical to the accelerations in the deployed configuration.

1.3.3.3.2 Deployment

1.3.3.3.2.1 Loads

During deployment, the mast structure must withstand the loads and deployed dynamic requirements defined for the orbital configuration.

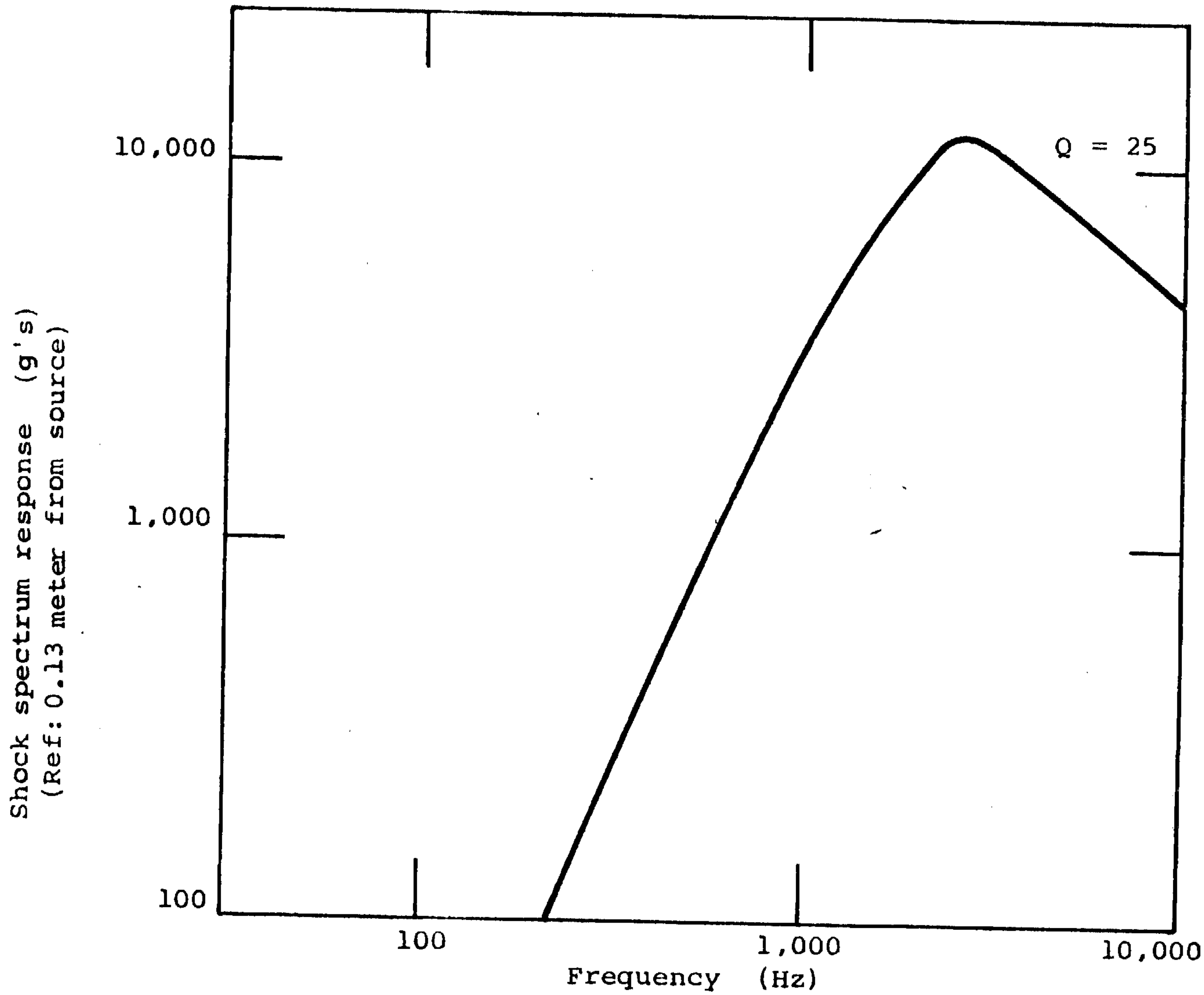


Figure B-3. - Pyrotechnic shock intensity

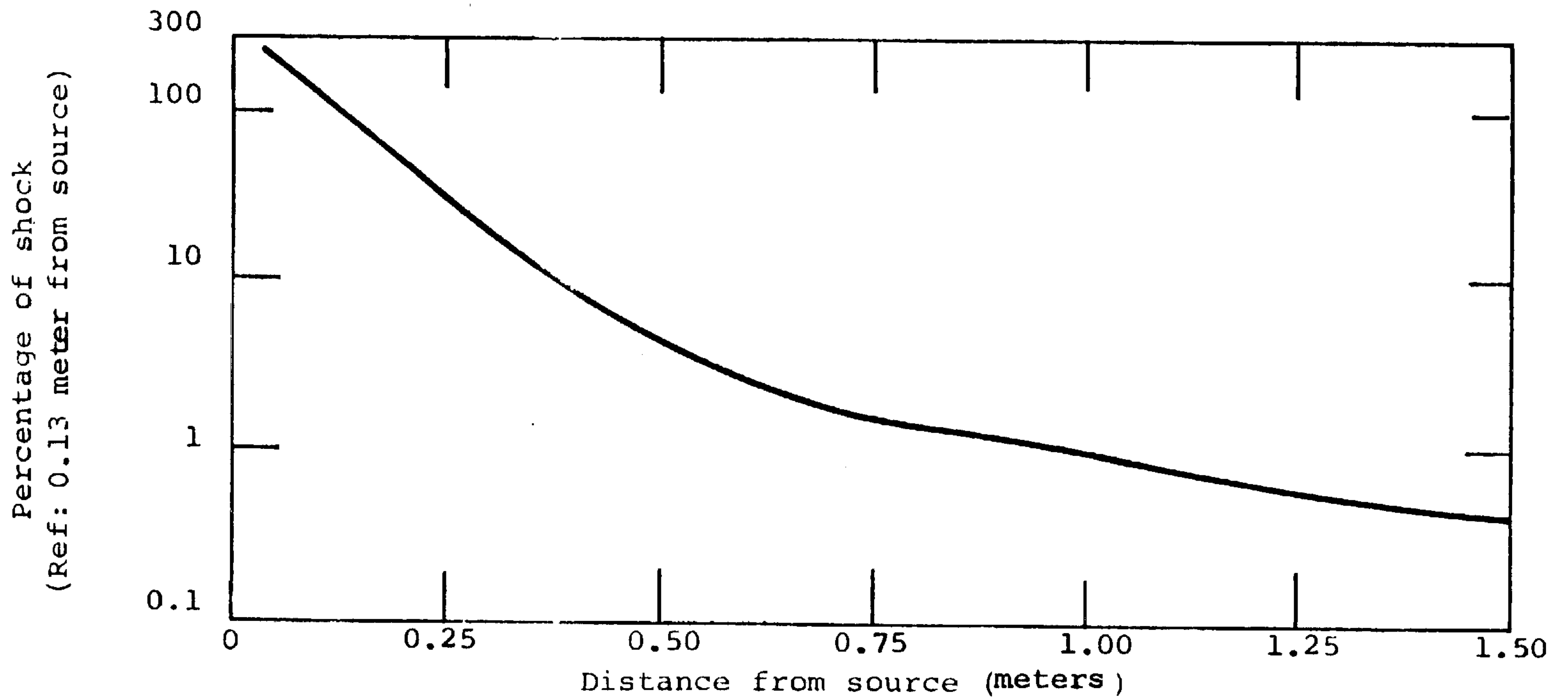


Figure B-4. - Shock attenuation vs equipment distance from source

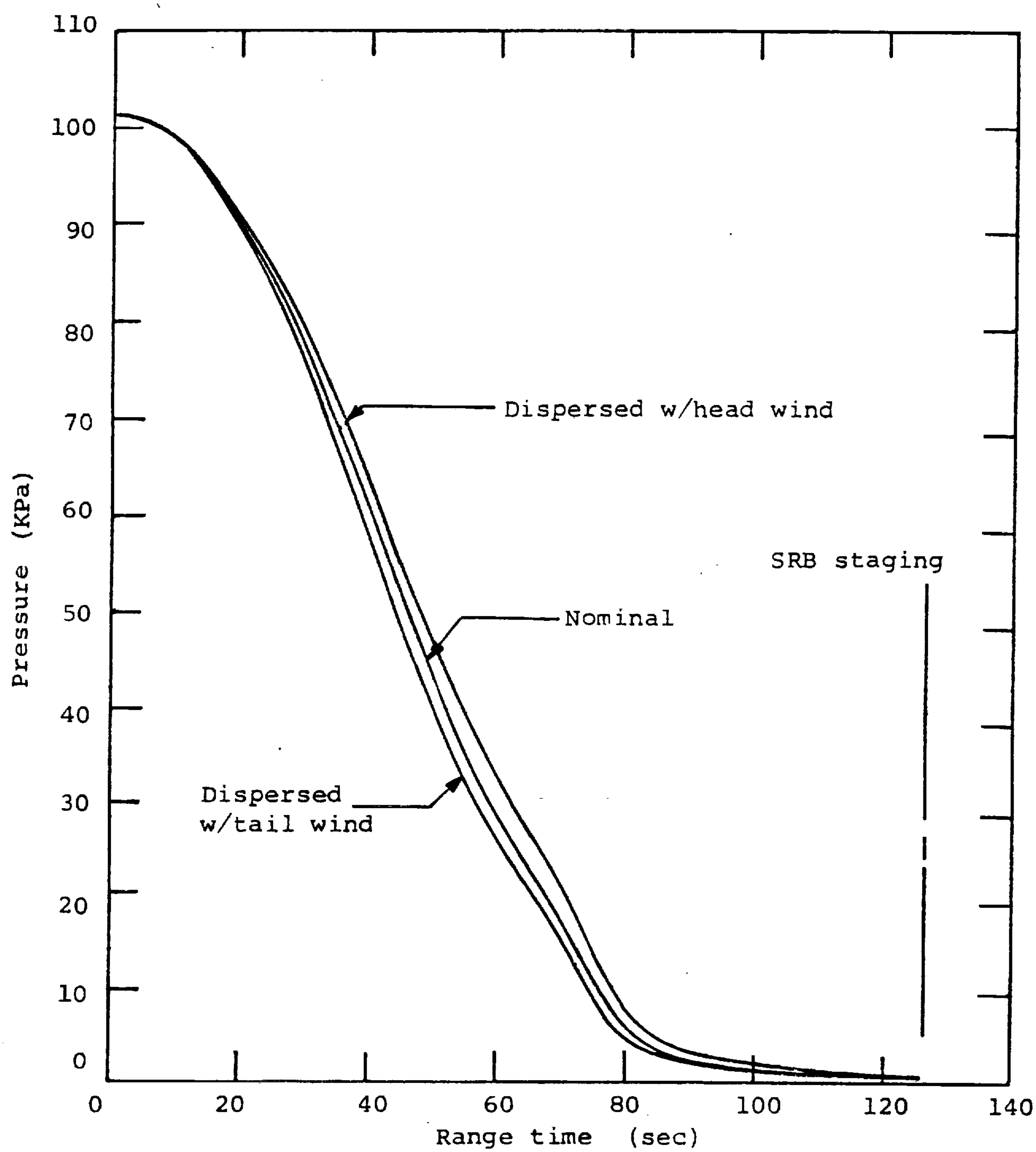


Figure B-5. - STS Orbiter payload bay internal pressure histories during ascent

1.3.3.3.2.2 Deployment Time

The maximum deployment time will be determined from mission requirements, and the minimum time from allowable deployment reaction forces on the spacecraft. An initial range of 1 to 10 minutes is assumed.

1.3.3.3.3 Orbital Configuration

Three structural requirements are identified for the orbital configuration: (1) STS Orbiter accelerations, (2) dynamic properties of the deployed mast and payload, and (3) forces due to gravity gradients, solar pressure, and aerodynamic drag. The dynamic properties of the support structure, especially large structures with low natural frequencies, are significant. A structure with a resonant frequency near the attitude control frequency can interact with the attitude control system and present significant problems.

1.3.3.3.3.1 Acceleration

The accelerations listed below are typical for the STS Orbiter, (reference 2).

Axis	Typical Rotational Accelerations
Roll	$\pm$ 0.032 deg/sec^2
Pitch	$\pm$ 0.026 deg/sec^2
Yaw	$\pm$ 0.023 deg/sec^2

Direction	Typical Translations
X	No acceleration
$\pm$ Y	0.275 cm/sec^2
$\pm$ Z	0.335 cm/sec^2

1.3.3.3.3.2 Dynamic Requirements

The mast system will be designed to satisfy the dynamic requirements defined in Figure B-6. Additional mission-peculiar dynamic requirements will be specified.

1.3.3.3.3.3 Gravity Gradients

The gravity-gradient forces are proportional to the mass of the systems and the square of their distances from the center of the earth. Deflections and loads produced by these forces must be within specified limits.

1.3.3.3.3.4 Solar Pressure

Typical solar-pressure forces are approximately 9×10^{-6} N/m^2. Deflections and loads produced by these forces must be within specified limits.

1.3.3.3.3.5 Aerodynamic Drag

Typical aerodynamic drag for a 100-nautical-mile orbit is 0.03 N/m^2. Deflections and loads produced by these forces must be within specified limits.

1.3.4 Interface Requirements

Interface requirements include mast-structure stowage envelope, power requirements, deployment-mechanism interface, payload and STS Orbiter interface. These requirements are not defined as part of this study.

1.3.5 Environments

The environmental requirements considered will be thermal, ground-handling loads, launch vibration, shock, pressure, and orbital acceleration. Other environment requirements such as storage temperature and humidity will not be evaluated as part of this study. Ground-handling loads, launch vibration, shock, pressure, and orbital accelerations have previously been defined.

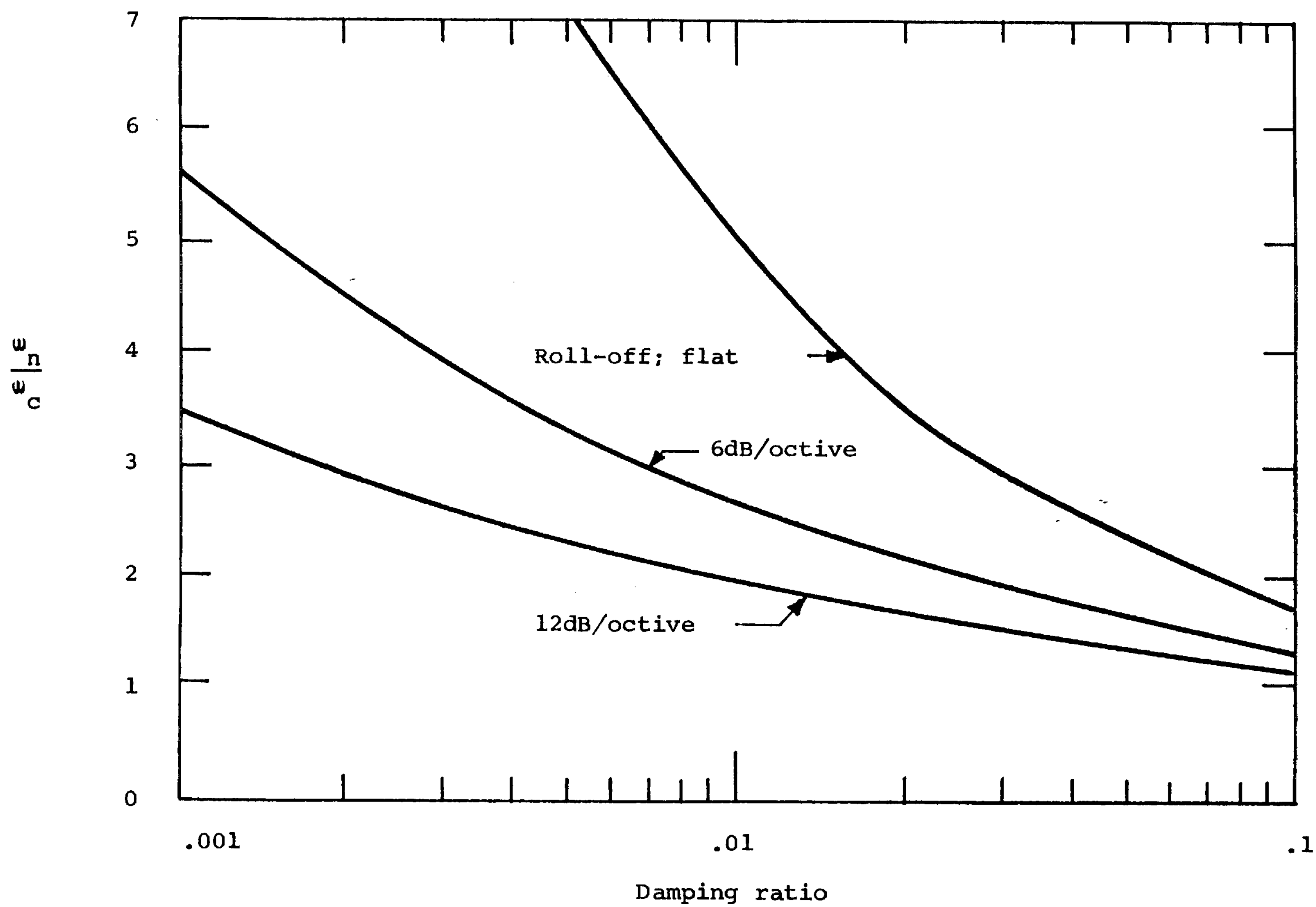


Figure B-6. - Required stiffness for stable control

1.3.5.1 Thermal Requirements

Worst-case temperature levels and gradients will be determined. Worst-case will be defined as a combination of temperatures and temperature gradients that produce the maximum precision error in the structure. Shadowing from the STS Orbiter, solar panels, payload, and the mast structure itself will be considered.

1.3.5.1.1 Solar Radiation

The solar radiation constant for a 100-nautical-mile orbit is 1399 W/m^2.

1.3.5.1.2 Earth Radiation

The Earth radiation constant for a 100-nautical-mile orbit is 243 W/m^2. Earth-thermal radiation is being considered since it limits component temperature excursions during earth shadow periods.

1.3.5.1.3 Earth Albedo

Earth albedo radiation for a 100-nautical-mile orbit is 30 percent.

1.3.5.1.4 Earth Shadow

The maximum earth shadow time is not specified for this study.

1.3.6 General Design Features

Design features will be considered which affect material properties, materials compatibility, thermal control properties, lifetime, cabling, interchangeability and replaceability, electromagnetic interference, and rf transparency.

1.3.7 Reliability

A typical reliability number for extension of deployable support structures is 0.99 or better.

1.3.8 Testing

It is very desirable for the deployable structure to be capable of testing in a 1 g environment because it reduces total costs and improves the reliability in space. This can be accomplished in a vertical orientation with gravity compensation, or horizontally on a low-friction surface.

APPENDIX C

ANALYSIS - ASTROMAST

This concept was designed to meet the following selected requirements:

Length .... 100 meters

Tip mass .... 315 kg

EI .... 2.87×10^{6} N-m^{2}

Bending strength .... ≥ 1 500 N-m

The selected configuration is a coilable lattice Astromast with

Diameter .... 1.12 meters

Longeron size .... 1.1 cm x 1.1 cm

Longeron material .... S-glass

The following design equations are derived from reference 1.

The bending stiffness is determined to be:

$$EI = 1.5\ (E)\ (A_{longeron})\ (R_{boom})^{2}$$

$$EI = 1.5 \times 5.0 \times 10^{10} \times (0.011 \times 0.011) \times \left(\frac{1.12}{2}\right)^{2} = 2.85 \times 10^{6}\ \text{N-m}^{2}$$

The maximum bending strength is determined by multiplying the buckling load of the longeron by the distance between the longerons:

$$M_{cr} = P_{cr} \times (1.12\ \text{m})$$

$$= \frac{\pi^2 \; 5.0 \times 10^{10} \; \frac{(0.011)^4}{12}}{1.2 \times \left(\frac{1.12}{2}\right)^2} \; (1.12)$$

$$= 1\ 500 \text{ N-m}$$

The weight of the Astromast is

$$W = 3 \times f \times \rho \times A_\ell \times L$$

$$= 3 \times 3.4 \times 1.94 \times 10^3 \times (0.011 \times 0.011) \times 100$$

$$= 239 \text{ kg}$$

The frequency of the Astromast with the 315 kg tip mass is

$$f = 0.16\sqrt{\frac{3EI}{(315 + 0.236\, w\ell)\, \ell^3}}$$

$$= 0.16\sqrt{\frac{2.85 \times 10^6}{(315 + 0.236 \times 239)\, 100^3}}$$

$$= 0.024 \text{ Hz}$$

APPENDIX D

ANALYSIS
COLUMN SUPPORTED BY TWO-LEVEL GUYLINES

This concept was designed to meet the following selected requirements:

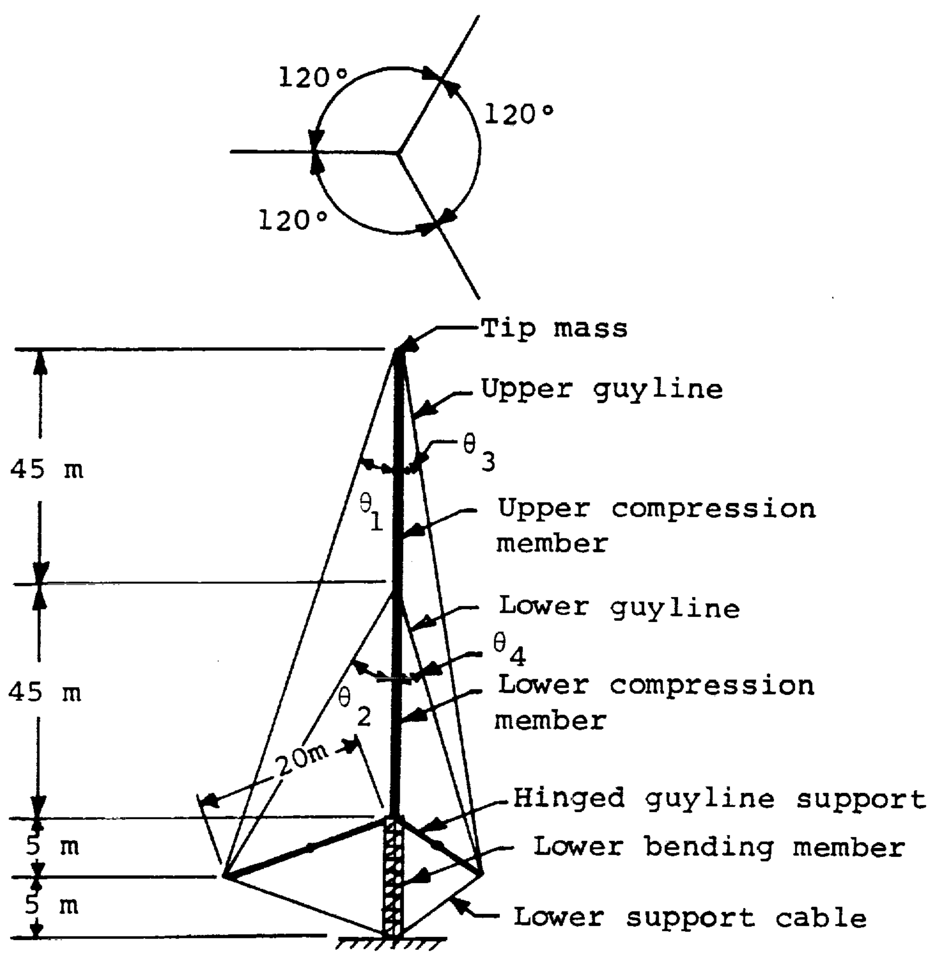
Length .... 100 meters

Tip mass .... 315 kg

EI .... 2.87×10^6 N-m^2

Bending strength .... $\geq$ 1 500 N-m

This concept is assumed to have the geometry shown below



The size of various members is based on the following criteria:

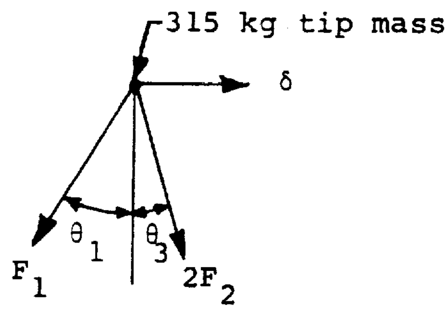
1. All guylines and lower support cables are steel. The central compression member is a continuous-longeron Astromast. The hinged guyline supports are graphite/epoxy tubes.

2. Tip mass = 315 kg

3. All structural resonant frequencies are greater than first mode frequency of baseline Astromast with 315 kg tip mass.

4. Capability of supporting lateral tip load greater than that of baseline Astromast.

The upper guylines are assumed to be 0.64 cm x 0.013 cm steel tape. The frequency of lateral vibration of the 315 kg tip mass is approximated by assuming that lateral motion is only restrained by the tapes, as shown below



$$\theta_1 = \tan^{-1}\left(\frac{\sqrt{20^2 - 5^2}}{95}\right) = 11.5°$$

$$\theta_3 = \tan^{-1}\left(\frac{\sqrt{20^2 - 5^2}\cos 60°}{95}\right) = 5.8°$$

The lateral stiffness k_1 is given by

$$k_1 = \frac{E A_1}{\ell_1} (\sin^2 11.5° + 2 \sin^2 5.8°)$$

Where A_1 and ℓ_1 are the area and the length, respectively, of the upper guyline

$$A_1 = 0.64 \times 0.013 = 0.0084 \text{ cm}^2$$

$$\ell_1 = \sqrt{20^2 - 5^2 + 95^2} = 97 \text{ m} \simeq 100 \text{ m}$$

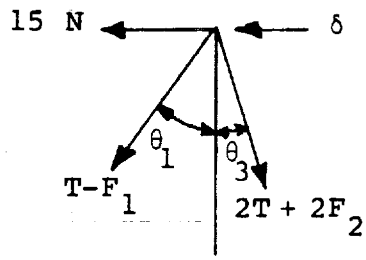
$$k_1 = \frac{2 \times 10^7 \times 0.0084}{10\ 000} (\sin^2 11.5° + 2 \sin^2 5.8°) = 1.0 \text{ N/cm} = 100 \text{ N/m}$$

The frequency of lateral vibration of the 315 kg tip mass is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = \frac{1}{2\pi}\sqrt{\frac{100}{315}} = 0.091 \text{ Hz}$$

Thus, the 0.64 cm x 0.013 cm steel tapes provide a substantially higher frequency than the frequency of the baseline Astromast.

The Astromast will sustain a bending moment of 1 500 N.m which corresponds to an approximate 15 N lateral tip load for a 100 meter mast. The tension in the upper guylines will be set in a manner that the unloaded guylines become slack under the action of a 15 N lateral tip load as shown below



$$F_1 = \frac{EA_1}{\ell_1} \delta \sin 11.5°$$

$$F_2 = \frac{EA_1}{\ell_1} \delta \sin 5.8°$$

$$15 \text{ N} = F_1 \sin 11.5° + 2F_2 \sin 5.8° = \frac{EA_1}{\ell_1} \delta (\sin^2 11.5° + 2 \sin^2 5.8°)$$

$$\delta = \frac{15 \ell_1}{EA_1 (\sin^2 11.5° + 2 \sin^2 5.8°)} = \frac{15 \times 10\ 000}{2 \times 10^7 \times 0.0084 (\sin^2 11.5° + 2 \sin^2 5.8°)}$$

$$\delta = 15 \text{ cm}$$

$$F_1 = \frac{2 \times 10^7 \times 0.0084 \times 15 \sin 11.5°}{10\ 000} = 50 \text{ N}$$

The pretension in the guyline must be at least 50 N to prevent unloading under this condition. For the reversed loading direction the tension in the guyline is 100 N and the stress is

$$\sigma = \frac{100 \text{ N}}{0.0084 \text{ cm}^2} = 12\ 000 \text{ N/cm}^2$$

The total weight of the upper guylines W_{ug} is

$$W_{ug} = 3 \times \ell_1 \times \rho \times A_1 = 3 \times 10\ 000 \times 0.0083 \times 0.0084 = 2.0 \text{ kg}$$

The fundamental frequency of the upper guyline is

$$f_1 = \frac{1}{2\ell_1}\sqrt{\frac{T_1}{\gamma_1}} = \frac{1}{2 \times 100}\sqrt{\frac{50}{0.0083 \times 0.0084 \times 100}} = 0.44 \text{ Hz}$$

The compression in the upper section F_{Cu} is given by

$$F_{Cu} = 3 \times 50 \cos 11.5° = 150 \text{ N}$$

The size of the upper section is established for a compressive load of 225 N (see compressive load calculated for lower section). Let I_u and ℓ_u be the moment of inertia and length of the upper member

$$225 = \frac{\pi^2 E I_u}{\ell_u^{\,2}} = \frac{\pi^2 \times 0.5 \times 10^7 \times I_u}{4\ 500^2} = 2.43\ I_u$$

$$I_u = 93 \text{ cm}^4$$

Assume the upper member is an 0.4-meter-diameter Astromast

$$EI = 1.5(E)(A_{longeron})(R_{boom})^2$$

therefore

$$A_\ell = \frac{93}{1.5(20)^2} = 0.155 \text{ cm}^2$$

therefore

$$W_u = 3 \times f \times \rho \times A_\ell \times L$$

$$= 3 \times 3.4 \times 1.94 \times 10^3 \times (0.0000155)45$$

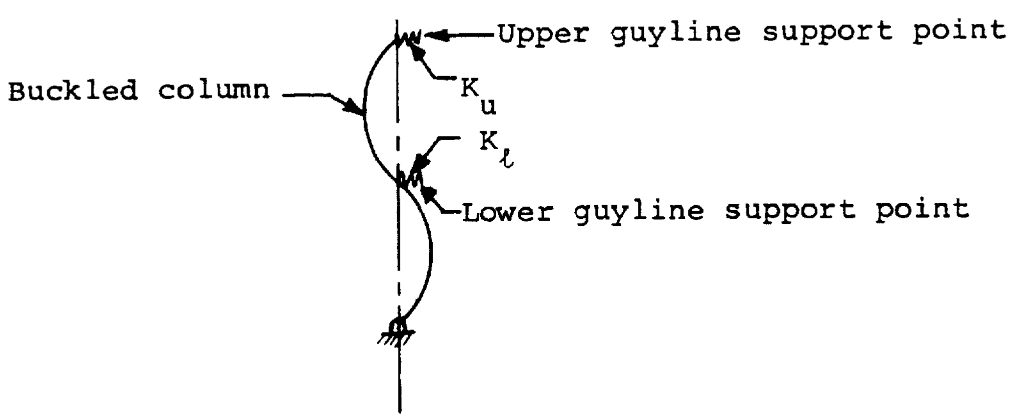
$$= 14 \text{ kg}$$

The fundamental frequency of the upper section (neglecting compressive load) is approximately

$$f = 0.16\sqrt{\frac{48\,EI}{0.486\,W_u \ell^3}} = 0.16\sqrt{\frac{48 \times 1 \times 10^7 \times 6\ 2 \times 100}{0.486 \times 14 \times 4\ 500^3}} = 0.36 \text{ Hz}$$

Even though this frequency is substantially reduced by the column compression, the frequency should nevertheless be substantially higher than the baseline fundamental frequency.

If the lateral stiffness at the guyline attachment points is sufficiently high, the first buckling mode of the structure will be as shown below



At this time it is assumed that the column has a uniform EI of 4.7×10^8 N cm^2. The required stiffness of the support points is given by

$$k_u = k_\ell = \frac{4m^3 \pi^2 EI}{\ell^3} = \frac{4 \times 2^3 \times \pi^2 \times 4.7 \times 10^8}{9\ 000^3} = 0.21 \text{ N/cm}$$

The upper guylines have been shown to provide a lateral stiffness of 1.0 N/cm. If the lower guylines are assumed to be 0.64 cm x 0.013 cm steel strip, they will provide a stiffness k_ℓ of

$$k_\ell = \frac{EA_2}{\ell_2} (\sin^2 \theta_2 + 2 \sin^2 \theta_4)$$

Where $A_2 = 0.64 \times 0.013 = 0.0084 \text{ cm}^2$, $\ell_2 = \sqrt{20^2 - 5^2 + 50^2} = 53.6$ m and θ_2 and θ_4 are given by

$$\theta_2 = \tan^{-1}\left(\frac{\sqrt{20^2 - 5^2}}{53.6}\right) = 19.9°$$

$$\theta_4 = \tan^{-1}\left(\frac{\sqrt{20^2 - 5^2}}{53.6} \cos 60°\right) = 10.2°$$

$$k_\ell = \frac{2 \times 10^7 \times 0.0084}{5\ 360} (\sin^2 19.9° + 2 \sin^2 10.2°) = 5.5 \text{ N/cm}$$

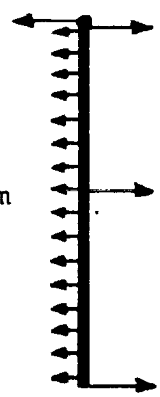
Lower guyline weight $W_{\ell g}$ is

$$W_{\ell g} = 3 \times \rho A \ell = 3 \times 0.0083 \times 0.0084 \times 5\ 360 = 1.1 \text{ kg}$$

Tension in the lower guylines is established by requiring that none of the lateral guylines become slack under the action of a lateral acceleration producing a lateral load of 15 N on the 315 kg tip mass. The mast is assumed to weigh 30 kg for this calculation. The lateral acceleration required for a 15 N lateral load at the tip mass is

$$a = \frac{F}{m} = \frac{15\ \text{N}}{315\ \text{kg}} = 0.048\ \text{m/sec}^2$$

The lateral load on the mast = 30 x 0.048 = 1.44 N. The forces required to balance the lateral loads will be assumed to be as shown below



The tension in lower guylines is calculated below.

$$0.7\ \text{N} = \frac{EA}{\ell}\,\delta\,(\sin^2 19.9° + 2\sin^2 10.2°)$$

$$\delta = \frac{5\ 360 \times 0.7}{2 \times 10^7 \times 0.0084\,(\sin^2 19.9° + 2\sin^2 10.2°)} = 0.13\ \text{cm}$$

$$F = \frac{EA}{\ell}\,\delta \sin 19.9° = \frac{2 \times 10^7 \times 0.0084 \times 0.13}{5\ 360} \sin 19.9° = 1.4\ \text{N}$$

The tension in the lower guylines is set at 1.4 N to prevent slackening. The compression force in the lower compression member $F_{C\ell}$ is given by

$$F_{C\ell} = 3 \times 50 \cos 11.5° + 3 \times 1.4 \cos 19.9° = 151 \text{ N}$$

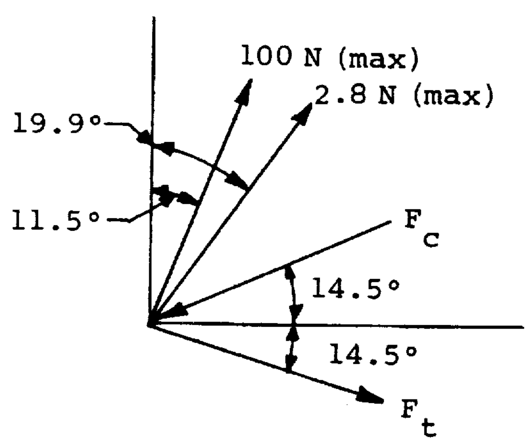
The compression in the lower section is set at 225 N to provide a factor of safety of 1.5 on Euler buckling. The weight W_ℓ of the 0.4-meter-diameter Astromast, which is 50 meters long, is

$$W_\ell = 15.6 \text{ kg}$$

The fundamental frequency of the lower guylines is

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{\gamma}} = \frac{1}{2 \times 53.6}\sqrt{\frac{1.4}{0.0083 \times 0.0084 \times 100}} = 0.13 \text{ Hz}$$

The loads in the hinged guyline support and lower support cable are shown in the following sketch.



$$F_c \sin 14.5° + F_t \sin 14.5° = 2.8 \cos 19.9° + 100 \cos 11.5°$$

$$F_c \cos 14.5° - F_t \cos 14.5° = 2.8 \sin 19.9° + 100 \sin 11.5°$$

$$0.25\, F_c + 0.25\, F_t = 100.6$$

$$0.97\, F_c - 0.97\, F_t = 20.9$$

$$F_c = 212 \text{ N}$$

$$F_t = 190 \text{ N}$$

The moment of inertia of the lower hinged guyline support I_n is calculated from Euler buckling

$$P = \frac{\pi^2 E\, I_n}{\ell_n^{\,2}}$$

$$2 \times 212 = \frac{\pi^2 \times 1.0 \times 10^7\, I_n}{2\ 000^2} = 24.7\ I_n$$

$$I_n = 17 \text{ cm}^4$$

Considering a thin-walled tube of diameter d_n and thickness $0.005\, d_n$

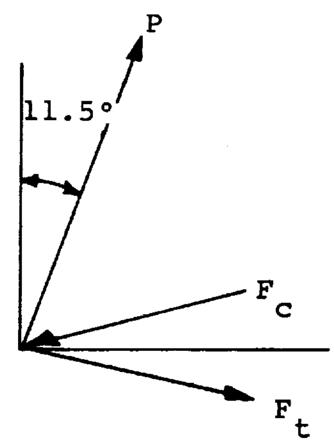
$$17 = \frac{\pi d^3 t}{8} = \frac{\pi \times 0.005\, d_n^{\,4}}{8}$$

$$d_n = \left(\frac{17 \times 8}{\pi \times 0.005}\right)^{0.25} = 9.6 \text{ cm}$$

Using d_n = 10 cm, and t_a = 0.05 cm, the total weight W_h of the three hinged guyline supports is

$$W_n = 3\rho A \ell = 3 \times 0.0017 \times \pi \times 10 \times 0.05 \times 2\,000 = 16 \text{ kg}$$

The lower support cable is assumed to be a 0.64 cm 0.013 cm tape. The effective EA of the upper guylines, including the effects of the lower support cable, is calculated below



$$0.25\,F_c + 0.25\,F_t = P\cos 11.5° = 0.98\,P$$

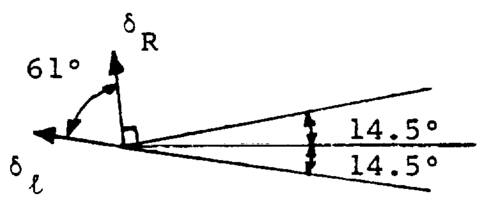
$$0.97\,F_t - 0.97\,F_t = P\sin 11.5° = 0.20\,P$$

$$F_t = 1.86\,P$$

The elongation δ_t is given by

$$\delta_t = \frac{F_t \ell}{AE} = \frac{1.86\ P \times 2\,000}{0.0084 \times 2 \times 10^7} = 0.022\ P$$

The end of the hinged guyline support translates δ_R resulting from an elongation δ_ℓ of the lower support cable.



$$\delta_R = \frac{\delta_\ell}{\cos 61°} = 2.06\ \delta_\ell = 0.045\ P$$

The total deflection at the upper guyline connection in the direction of the guyline is given by

$$\delta = 0.045\ P \cos 26° + \frac{P \times 9\,695}{0.0084 \times 2 \times 10^7} = 0.040\ P + 0.058\ P$$

where

0.040 P is from lower support cable stretch
0.058 P is from upper guyline stretch

This stretch of the lower support cables reduces the effective stiffness of the upper guylines by approximately 40%. Since this reduction is very large for the relatively short length of lower support cables, the area of the lower support cables will be increased by a factor of 5 - giving the lower support cables a stored volume approximately equal to that of the upper guylines. δ is now given by

$$\delta = \frac{0.040\,P}{5} + 0.058\,P = 0.008\,P + 0.058\,P$$

Now, stretch in the lower support cables only decreases the effective stiffness of the upper guylines by approximately 12%. The area of the lower support cables is

$$A = 0.0084 \times 5 = 0.042 \text{ cm}^2$$

The total weight of the three lower tension members $W_{\ell t}$ is

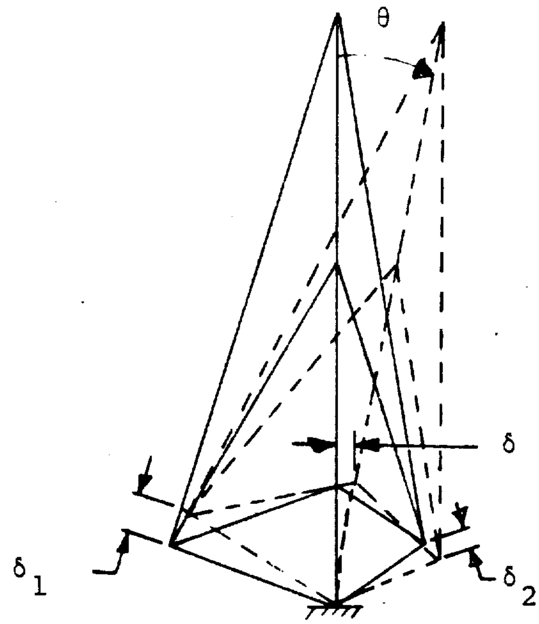
$$W_{\ell t} = 3\,P\,A\,\ell = 3 \times 0.0083 \times 0.042 \times 2\,000 = 2.1 \text{ kg}$$

Compression of the hinged guyline supports reduces the effective guyline stiffnesses. The EA of these members is

$$EA = E\pi dt = 10^7 \times \pi \times 10 \times 0.05 = 1.6 \times 10^7 \text{ N}$$

This is approximately 20 times higher than the EA of the lower support cables, therefore, deflection of the hinged compression members is negligible.

Since the lower bending member is expected to be substantially more stiff than the lower compression member, it is assumed that no moment is transmitted from the lower bending member to the lower compression section. The deformed shape of the structure, resulting from bending of the lower member, would appear as sketched below.



Since the length of the lower bending member is essentially unchanged for small δ, the motion of the structure (with the exception of the lower bending member) will be a rigid-body rotation about the fixed base.

The moment of inertia of the structure for rotation about the fixed base is given by

$$I = \int x^2 dm$$

where x is the distance from the differential mass to the fixed base.

The components of the above integral are calculated below

$$U_m = \int x^2 dm = 315 \times 100^2 = 3.15 \times 10^6 \text{kg m}^2$$

$$C_M = \int x^2 dm = 30\left[\frac{1}{12}(90)^2 + 55^2\right] = 1.11 \times 10^5 \text{ kg m}^2$$

$$H_{GS} = \int x^2 dm = 16 \left[\frac{1}{12}(20)^2 + (10^2 + 7.5^2)\right] = 3.03 \times 10^3 \text{ kg m}^2$$

$$U_g = \int x^2 dm = 2 \left[\frac{1}{12}(97)^2 + (52.5^2 + 10^2)\right] = 7.3 \times 10^3 \text{ kg m}^2$$

$$L_g = \int x^2 dm = 1.1 \left[\frac{1}{12}(54)^2 + (30^2 + 10^2)\right] = 1.4 \times 10^3 \text{ kg m}^2$$

$$L_{SC} = \int x^2 dm = 2.1 \left[\frac{1}{12}(20)^2 + (10)^2\right] = 2.8 \times 10^2 \text{ kg m}^2$$

$$\text{Sum} = \int x^2 dm = 3.15 \times 10^6 + 1.11 \times 10^5 + 3.03 \times 10^3 + 1.4 \times 10^3 + 7.3 \times 10^3 + 2.8 \times 10^2 = 3.3 \times 10^6 \text{ kg m}^2$$

where

U_m = Tip mass

C_M = Compression members

H_{GS} = Hinged guyline support

U_g = Upper guylines

L_g = Lower guylines

L_{SC} = Lower support cable

The deflection δ of the end of the lower bending member is given by:

$$\delta = 10\,\theta = \frac{p \times 10^3}{3\,EI_\ell}$$

where EI_ℓ is the bending stiffness of the lower bending member. The torque about the fixed base is given by

$$T = 10\,p$$

Therefore,

$$10 \times \theta = \frac{T \times 10^2}{3\,EI_\ell}$$

$$\theta = \frac{10}{3\,EI_\ell}\,T$$

$$K_\theta = \frac{3\,EI_\ell}{10}$$

The frequency of oscillation about the fixed point is

$$f = \frac{1}{2\pi}\sqrt{\frac{K_\theta}{\int x^2 dm}} = \frac{1}{2\pi}\sqrt{\frac{3\,EI_\ell}{10 \times 3.3 \times 10^6}}$$

This formula neglects the mass of the lower bending member. The actual lower bending-member bending mode will also have a lower frequency than the assumed mode, because the actual mode will involve some amount of guyline stretch, etc. Consequently, the desired value of f will be taken as 0.05 Hz to allow for these unconservative approximations.

$$EI_\ell = \frac{3.3 \times 10^6 \times 10 \times (2\pi \times 0.05)^2}{3} = 1.1 \times 10^6 \text{ N-m}^2$$

Based on the requirements for the lower bending member of $EI = 1.1 \times 10^6$ N-m^2 and $M_{cr} = 1\,500$ N-m, the truss will be 1.0 m in diameter and have a weight-per-unit-length of 1.8 kg/m^2.

$$W_B = 1.80 \times 10 = 18 \text{ kg}$$

The total weight of the structure, not including the deployment mechanism, is

$$W_{total} = W_{ug} + W_u + W_{\ell g} + W_{\ell} + W_h + W_{\ell t} + W_B$$

$$= 2 + 14 + 1.1 + 15.6 + 16 + 2.1 + 18$$

$$= 68.8 \text{ kg}$$

APPENDIX E

ANALYSIS
COLUMN SUPPORTED BY LATTICE GUYLINES

This concept was designed to meet the following selected requirements:

Length .... 100 m

Tip mass .... 315 kg

EI .... 2.87×10^6 N-m^2

Bending strength .... ≥1 500 N-m

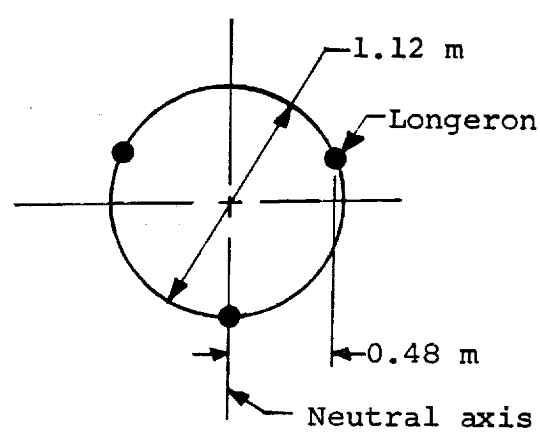
Mast diameter .... 1.12 m

Bay length .... 0.66 m

The three longerons with area A_L are assumed to be made of steel wire. They are sized from the relationship

$$I = \frac{2.87 \times 10^6}{E} = \frac{2.87 \times 10^{10}}{2 \times 10} = 1\ 435\ \text{cm}^4 = \sum A_L d^2 = 2 \times 0.48^2 A_L$$

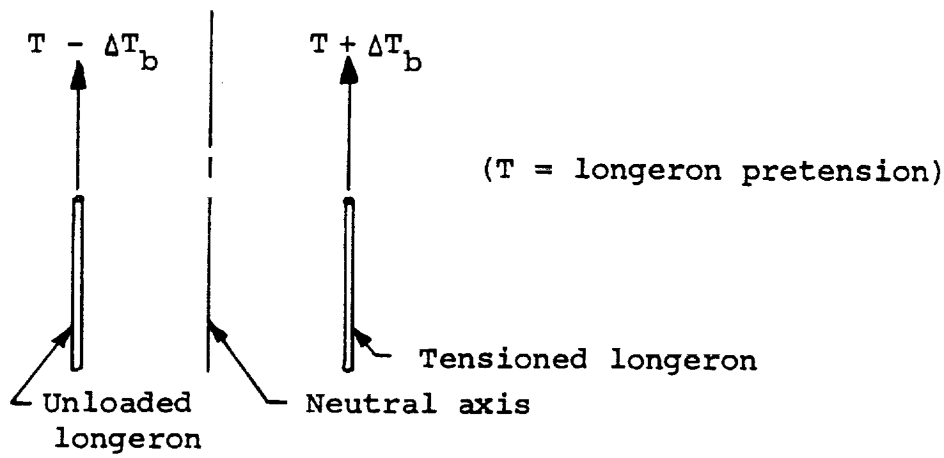
$$A_L = 0.30\ \text{cm}^2$$



The weight of the longeron W_L is

$$W_L = 0.30\ cm^2 \times 3 \times 0.0083\ kg/cm^3 \times 10\ 000\ cm = 74\ kg$$

The pre-tension T in the longerons is calculated based on the assumption that, under the action of an applied bending moment, the stress in the tensioned longeron approaches the working stress when the unloading longeron becomes slack.



The unloading longeron becomes slack when $\Delta T_b = T$. Therefore

$$\sigma_{max} = \frac{T + \Delta T_b}{A_L} = \frac{2T}{A_L}$$

For a working stress of 20 000 N/cm^2

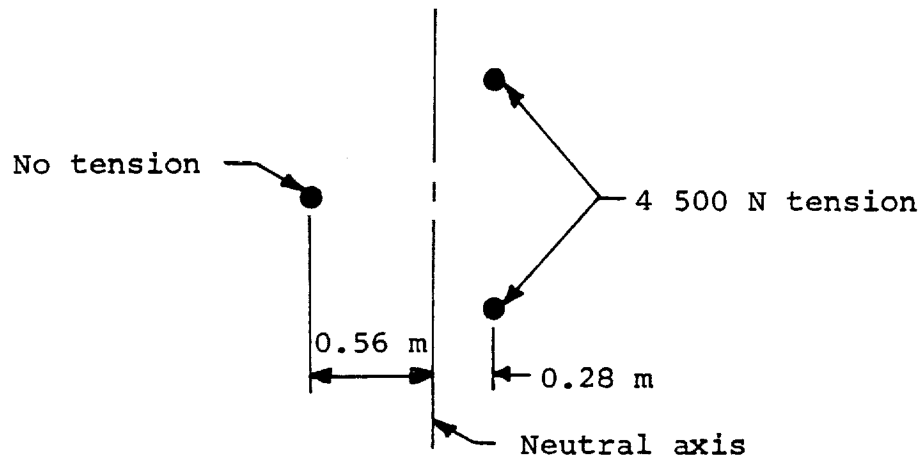
$$T = \frac{\sigma_{max} A_L}{2} = \frac{20\ 000 \times 0.3}{2} = 3\ 000\ N$$

The ultimate moment is obtained when two longerons are tensioned to 4 500 N and a single longeron is unloaded (becomes slack) by 3 000 N, therefore, as shown in the following sketch

$$M_{cr} = 1\ 500 \times 2 \times 0.28 + 3\ 000 \times 0.56 = 2\ 520\ N\text{-}m$$

or

$$M_{cr} = 4\ 500 \times 0.28 \times 2 = 2\ 520\ N\text{-}m$$



A 3.4 cm diameter stainless-steel BI-STEM was selected for the central element. This BI-STEM has a cross-sectional area of $0.36\ cm^2$, a minimum EI of $950\ N\text{-}m^2$ and a thickness of 0.018 cm. The stress in the element for a compressive load of 9 000 N is $\sigma_{BI\text{-}STEM} = 9\ 000/0.36 = 25\ 000\ N/cm^2$.

The ultimate stress in the element is $100\ 000\ N/cm^2$ and the local crippling stress σ_{crip} is given by

$$\sigma_{crip} = \frac{Et}{d\sqrt{3(1-\mu^2)}} = \frac{2 \times 10^7 \times 0.018}{3.4\sqrt{3(1-0.3^2)}} = 65\ 000\ N/cm^2$$

Since the element is supported by a batten frame every 0.66 meter, the Euler buckling load is given by

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} = \frac{\pi^2 \times 950}{0.66^2} = 21\ 500\ N$$

The weight of the BI-STEM element $W_{BI-STEM}$ is

$$W_{BI-STEM} = 0.36\ cm^2 \times 0.0083\ kg/cm^3 \times 10\ 000\ cm = 30\ kg$$

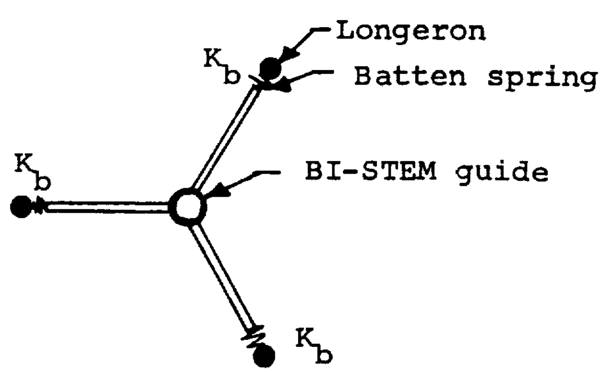
In order for the battens to provide effective restraint against BI-STEM buckling, they must have a lateral stiffness constant given by the formula

$$k_{lat} \geqq \frac{4m^3\pi^2 EI}{\ell^3} = \frac{4 \times 151^3 \times \pi^2 \times 950}{100^3 \times 100} = 1\ 300\ N/cm$$

Where m is the number of bays

$$m = \frac{100}{0.66} = 151\ bays$$

The batten frames are assumed to be constructed as shown in the sketch below

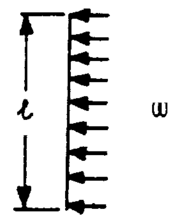


The stiffness of the batten springs K_b is given by

$$K = 1\ 300\ N/cm = 1.5\ K_b$$

$$K_b = 870\ N/cm$$

The batten springs must be compressed to provide tension in the diagonals. The required tension in the diagonals is calculated by assuming that one diagonal element becomes slack simultaneously with the attainment of ultimate bending moment for the condition of uniform lateral load as shown below



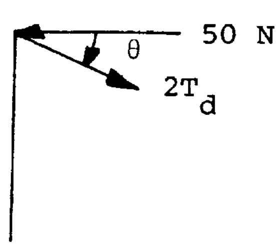
Shear = $\omega\ell$

$$\text{Moment} = \frac{\omega\ell^2}{2}$$

$$\omega = \frac{2 \times 2\ 600}{100^2} = 0.5 \text{ N/m}$$

Shear = 0.5 x 100 = 50 N

It will be assumed that this shear load must be removed by the shear panel aligned with the load (the contributions of the oblique shear panels will be ignored). When the unloaded diagonal becomes slack the tensioned diagonal must support the lateral load T_d as shown below

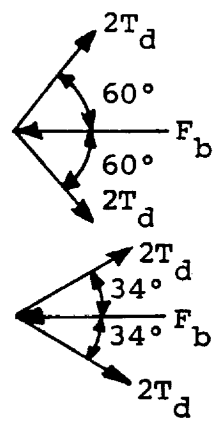


$$\theta = \tan^{-1}\frac{0.66}{0.96} = 34°$$

$$2T_d \cos 34° = 50$$

$$T_d = 30 \text{ N}$$

The batten load F_b required for a diagonal load of 30 N is given by



$$F_b = 4T_d \cos 60° \cos 34° = 50 \text{ N}$$

The required batten spring deflections δ for batten compression of 50 N is

$$\delta = \frac{50 \text{ N}}{87° \text{ N/cm}} = 0.057 \text{ cm}$$

It is assumed that batten lengths can only be manufactured to a tolerance of ±0.025 cm, therefore, the nominal deflection required to ensure a minimum force of 50 N is $\delta = 0.025 + 0.057 = 0.082$ cm, and the peak deflection is $\delta = 0.11$ cm. This deflection produces a batten compression load of

$$F_b = 0.11 \times 870 = 96 \text{ N}$$

The batten members are assumed to be tubular graphite epoxy with d/t = 10. The diameter required to prevent lateral buckling is given by

$$P = 2(FS) \times 96 \simeq \frac{\pi^2 EI}{\ell^2} = \frac{\pi^2 \times 1 \times 10^7 \times 0.2R^4}{112^2}$$

$$R = 0.70 \text{ cm}$$

For a diameter of 1.4 cm, the weight per unit length is

$$\omega = 0.0017 \times \pi dt = 0.0017 \times \pi \times 1.4 \times 0.14 \times 100 = 0.10 \text{ kg/m}$$

and the total batten weight W_b is

$$W_b = 0.10 \times 151 \times 3 \times 0.56 = 25 \text{ kg}$$

The diagonals are sized to provide adequate lateral stiffness for the battens. Since the lateral stiffness of the batten must be 1 300 N/cm, the lateral stiffness at the battens provided by the diagonals will be assumed to be k_d = 10 x 1 300 = 13 000 N/cm to ensure adequate support. The area of the steel wire diagonal A_d is then calculated to be

$$K_d = \frac{2E A_d \cos^2 34°}{\ell_d} = 13\ 000$$

$$A_d = \frac{13\ 000 \sqrt{97^2 + 61^2}}{2 \times 2 \times 10^7 \times \cos^2 34°} = 0.054 \text{ cm}^2$$

The diagonal weight W_d is given by

$$W_d = \sqrt{97^2 + 61^2} \times 6 \times 151 \times 0.054 \times 0.0083 = 47 \text{ kg}$$

The weight of the batten BI-STEM guides will be assumed to be 0.05 kg/guide and the weight of the longeron-batten-diagonal joints will be assumed to be 0.05 kg/joint. The total system weight W_s (excluding deployment mechanism) is given by

$$W_s = W_{BI\text{-}STEM} + W_L + W_d + W_b + W_{guide} + W_{joint}$$

$$= 30 + 74 + 47 + 25 + 151 \times 0.05 + 151 \times 3 \times 0.05$$

$$= 30 + 74 + 47 + 25 + 8 + 23 = 207 \text{ kg}$$

REFERENCES

1. Crawford, R. F.: Strength and Efficiency of Deployable Booms for Space Applications, Astro Research Corporation paper presented at AAS/AIAA Variable Geometry and Expandable Structures Conference, April 1971, AIAA Preprint No. 71.

2. NASA Johnson Space Center: Space Shuttle Program. Space Shuttle System Payload Accommodations. Level II Program Definition and Requirements. JSC 07700, Volume XIV, Revision D, November 26, 1975.